Unit 9.7 Inverse Functions NOTES

In particular, for this pair of functions,

$$f(g(2)) = 2 \quad \text{and} \quad g(f(2)) = 2.$$

In fact, for any value of x,

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x,$$

or $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

Because of this property, g is called the *inverse* of f.

Inverse Function

Let f be a one-to-one function. Then g is the **inverse function** of f if

$(f \circ g)(x) = x$ for every x in the domain of g,

and

$(g \circ f)(x) = x$ for every x in the domain of f.

DECIDING WHETHER TWO FUNCTIONS ARE INVERSES

Let functions f and g be defined by $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$, respectively. Is g the inverse function of f?

Solution

$$(f \circ g)(x) = f(g(x)) = \left(\sqrt[3]{x+1}\right)^3 - 1$$

$$= x + 1 - 1$$

$$= x$$

AND

Let functions f and g be defined by $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$, respectively. Is g the inverse function of f?

Solution

$$(g \circ f)(x) = g(f(x)) = \sqrt[3]{(x^3 - 1) + 1}$$

$f(x) = x^3 - 1;$
$g(x) = \sqrt[3]{x+1}$

$$= \sqrt[3]{x^3}$$

$$= x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, function g is the inverse of function f.

Finding the Equation of the Inverse of $y = f(x)$

For a one-to-one function f defined by an equation $y = f(x)$, find the defining equation of the inverse as follows. (You may need to replace $f(x)$ with y first.)

Step 1 Interchange x and y.

Step 2 Solve for y.

Step 3 Replace y with $f^{-1}(x)$.

FINDING EQUATIONS OF INVERSES

Example:

Solution

$$y = 2x + 5 \qquad y = f(x)$$

$$x = 2y + 5 \qquad \text{Interchange } x \text{ and } y.$$

$$2y = x - 5 \qquad \text{Solve for } y.$$

$$y = \frac{x-5}{2}$$

$$f^{-1}(x) = \frac{1}{2}x - \frac{5}{2} \qquad \text{Replace } y \text{ with } f^{-1}(x).$$

Another example:

$$f(x) = (x-2)^3$$

$$y = (x-2)^3 \qquad \text{Replace } f(x) \text{ with } y.$$

$$x = (y-2)^3 \qquad \text{Interchange } x \text{ and } y.$$

$$\sqrt[3]{x} = \sqrt[3]{(y-2)^3} \qquad \text{Take the cube root on each side.}$$

$$\sqrt[3]{x} = y - 2$$

$$\sqrt[3]{x} + 2 = y \qquad \text{Solve for } y.$$

$$f^{-1}(x) = \sqrt[3]{x} + 2 \qquad \text{Replace } y \text{ with } f^{-1}(x).$$