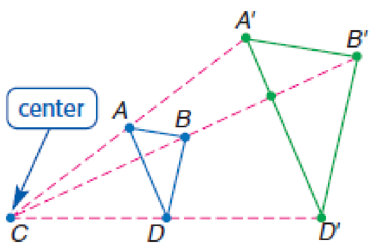
Notes 9.5 Dilations

CLASSIFY DILATIONS All of the transformations you have studied so far in this chapter produce images that are congruent to the original figure. A dilation is another type of transformation.

A **dilation** is a transformation that may change the size of a figure. A dilation requires a center point and a scale factor. The figures below show how dilations can result in a larger figure and a smaller figure than the original.

$r = 2$



Triangle $A'B'D'$ is a dilation of $\triangle ABD$.

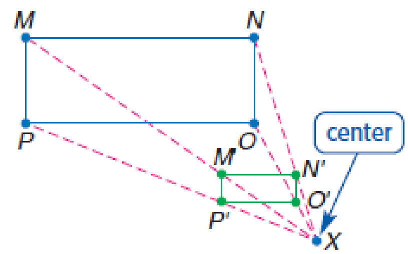
$CA' = 2(CA)$

$CB' = 2(CB)$

$CD' = 2(CD)$

$\triangle A'B'D'$ is larger than $\triangle ABD$.

$r = \frac{1}{3}$



Rectangle $M'N'O'P'$ is a dilation of rectangle $MNOP$.

$XM' = \frac{1}{3}(XM)$ $XN' = \frac{1}{3}(XN)$

$XO' = \frac{1}{3}(XO)$ $XP' = \frac{1}{3}(XP)$

Rectangle $M'N'O'P'$ is smaller than rectangle $MNOP$.

The value of r determines whether the dilation is an enlargement or a reduction.

Key Concept — Dilation

If $|r| > 1$, the dilation is an enlargement.

If $0 < |r| < 1$, the dilation is a reduction.

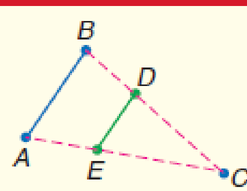
If $|r| = 1$, the dilation is a congruence transformation.

As you can see in the figures on the previous page, dilation preserves angle measure, betweenness of points, and collinearity, but does not preserve distance. That is, dilations produce similar figures. Therefore, a dilation is a **similarity transformation**.

This means that $\triangle ABD \sim \triangle A'B'D'$ and $\square MNOP \sim \square M'N'O'P'$. This implies that $\frac{A'B'}{AB} = \frac{B'D'}{BD} = \frac{A'D'}{AD}$ and $\frac{M'N'}{MN} = \frac{N'O'}{NO} = \frac{O'P'}{OP} = \frac{M'P'}{MP}$. The ratios of measures of the corresponding parts is equal to the absolute value scale factor of the dilation, $|r|$. So, $|r|$ determines the size of the image as compared to the size of the preimage.

Theorem 9.1

If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$.



Notes 9.5 Dilations Continued

Example 1 Determine Measures Under Dilations

Find the measure of the dilation image $\overline{A'B'}$ or the preimage $\overline{AB}$ using the given scale factor.

a. $AB = 12, r = -2$

$A'B' = |r|(AB)$

$A'B' = 2(12)$ $|r| = 2, AB = 12$

$A'B' = 24$ Multiply.

b. $A'B' = 36, r = \frac{1}{4}$

$A'B' = |r|(AB)$

$36 = \frac{1}{4}(AB)$ $A'B' = 36, |r| = \frac{1}{4}$

$144 = AB$ Multiply each side by 4.

When the scale factor is negative, the image falls on the opposite side of the center than the preimage.

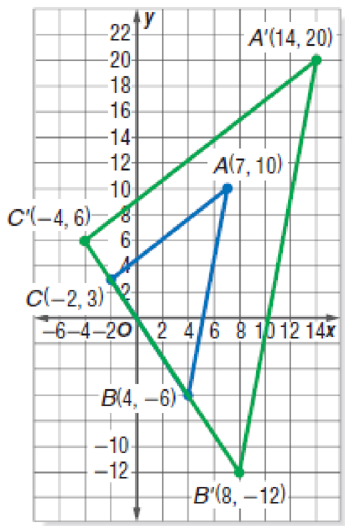
Theorem 9.2

If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$.

Example 3 Dilations in the Coordinate Plane

COORDINATE GEOMETRY Triangle ABC has vertices $A(7, 10)$, $B(4, -6)$, and $C(-2, 3)$. Find the image of $\triangle ABC$ after a dilation centered at the origin with a scale factor of 2. Sketch the preimage and the image.

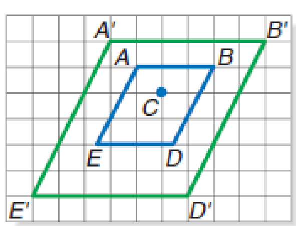
Preimage (x, y)	Image $(2x, 2y)$
$A(7, 10)$	$A'(14, 20)$
$B(4, -6)$	$B'(8, -12)$
$C(-2, 3)$	$C'(-4, 6)$



Example 4 Identify Scale Factor

Determine the scale factor for each dilation with center C. Then determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

a.



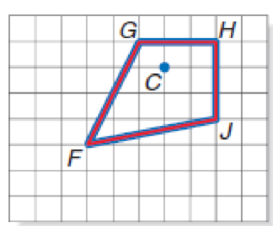
$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}}$$

$= \frac{6 \text{ units}}{3 \text{ units}}$ ← image length / ← preimage length

$= 2$ Simplify.

Since the scale factor is greater than 1, the dilation is an enlargement.

b.



$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}}$$

$= \frac{4 \text{ units}}{4 \text{ units}}$ ← image length / ← preimage length

$= 1$ Simplify.

Since the scale factor is 1, the dilation is a congruence transformation.