# 9-4 Operations with Functions

Warm Up

Lesson Presentation

Lesson Quiz

## Warm Up

**Simplify. Assume that all expressions are defined.**

**1.** $(2x + 5) - (x^2 + 3x - 2)$  $-x^2 - x + 7$

**2.** $(x - 3)(x + 1)^2$  $x^3 - x^2 - 5x - 3$

**3.** $\dfrac{x^2 - x - 6}{x^2 - 4}$  $\dfrac{x - 3}{x - 2}$

## Objectives

Add, subtract, multiply, and divide functions.

Write and evaluate composite functions.

## Vocabulary

composition of functions

You can perform operations on functions in much the same way that you perform operations on numbers or expressions. You can add, subtract, multiply, or divide functions by operating on their rules.

| Notation for Function Operations | |
|---|---|
| **Operation** | **Notation** |
| Addition | $(f+g)(x) = f(x) + g(x)$ |
| Subtraction | $(f-g)(x) = f(x) - g(x)$ |
| Multiplication | $(fg)(x) = f(x) \cdot g(x)$ |
| Division | $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$ |

## Check It Out! Example 1a

**Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.**

**$(f + g)(x)$**

$(f + g)(x) = f(x) + g(x)$

$= (5x - 6) + (x^2 - 5x + 6)$ *Substitute function rules.*

$= x^2$ *Combine like terms.*

## Check It Out! Example 1b

**Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.**

**$(f - g)(x)$**

$(f - g)(x) = f(x) - g(x)$

$= (5x - 6) - (x^2 - 5x + 6)$ *Substitute function rules.*

$= 5x - 6 - x^2 + 5x - 6$ *Distributive Property*

$= -x^2 + 10x - 12$ *Combine like terms.*

When you divide functions, be sure to note any domain restrictions that may arise.

## Check It Out! Example 2a

**Given $f(x) = x + 2$ and $g(x) = x^2 - 4$, find each function.**

**$(fg)(x)$**

$(fg)(x) = f(x) \bullet g(x)$

$= (x + 2)(x^2 - 4)$ *Substitute function rules.*

$= x^3 + 2x^2 - 4x - 8$ *Multiply.*

## Check It Out! Example 2b

$$\left(\frac{g}{f}\right)(x)$$

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$$

$$= \frac{x^2 - 4}{x + 2}$$ *Set up the division as a rational expression.*

$$= \frac{(x - 2)(x + 2)}{x + 2}$$ *Factor completely. Note that $x \neq -2$.*

$$= \frac{(x - 2)\cancel{(x + 2)}}{\cancel{(x + 2)}}$$ *Divide out common factors.*

$$= x - 2, \text{ where } x \neq -2$$ *Simplify.*

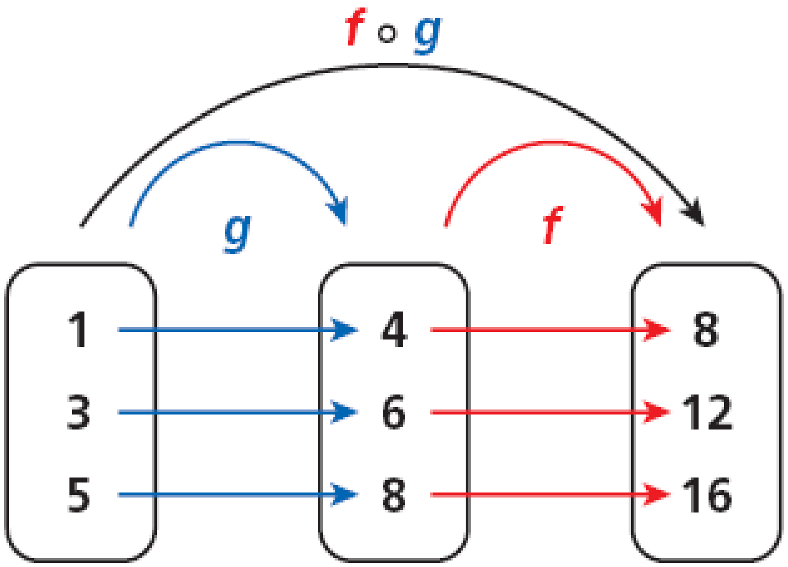
Another function operation uses the output from one function as the input for a second function. This operation is called the **composition of functions**.

## Composition of Functions

The composition of functions $f$ and $g$ is notated

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ is all values of $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.




To find $(f \circ g)(1)$, first find $g(1)$.

$$g(1) = 4$$

Then use 4 as the input into $f$:

$$f(4) = 8$$

So $(f \circ g)(1) = f(g(1)) = 8$.

The order of function operations is the same as the order of operations for numbers and expressions. To find *f*(*g*(3)), evaluate *g*(3) first and then substitute the result into *f*.

**Reading Math**

The composition $(f \circ g)(x)$ or $f(g(x))$ is read "*f* of *g* of *x*."

**Caution!**

Be careful not to confuse the notation for multiplication of functions with composition

$fg(x) \neq f(g(x))$

## Check It Out! Example 3a

**Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.**

**$f(g(3))$**

**Step 1** Find $g(3)$

$g(3) = 3^2$ $\quad$ $g(x) = x^2$

$= 9$

**Step 2** Find $f(9)$

$f(9) = 2(9) - 3$ $\quad$ $f(x) = 2x - 3$

$= 15$

So $f(g(3)) = 15$.

## Check It Out! Example 3b

**Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.**

**$g(f(3))$**

**Step 1** Find $f(3)$

$f(3) = 2(3) - 3$ $\quad f(x) = 2x - 3$

$= 3$

**Step 2** Find $g(3)$

$g(3) = 3^2$ $\quad g(x) = x^2$

$= 9$

So $g(f(3)) = 9$.

You can use algebraic expressions as well as numbers as inputs into functions. To find a rule for $f(g(x))$, substitute the rule for $g$ into $f$.

## Check It Out! Example 4a

**Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.**

**$f(g(x))$**

$f(g(x)) = 3(\sqrt{x} + 2) - 4$ *Substitute the rule g into f.*

$= 3\sqrt{x} + 6 - 4$ *Distribute. Note that $x \geq 0$.*

$= 3\sqrt{x} + 2$ *Simplify.*

The domain of $f(g(x))$ is $x \geq 0$ or $\{x|x \geq 0\}$.

## Check It Out! Example 4b

**Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.**

**$g(f(x))$**

$g(f(x)) =$ *Substitute the rule f into g.*

$=$ *Note that $x \geq \frac{4}{3}$.*

The domain of $g(f(x))$ is $x \geq \frac{4}{3}$ or $\{x | x \geq \frac{4}{3}\}$.

Composite functions can be used to simplify a series of functions.

## Check It Out! Example 5

**During a sale, a music store is selling all drum kits for 20% off. Preferred customers also receive an additional 15% off.**

**a.** Write a composite function to represent the final cost of a kit for a preferred customer that originally cost *c* dollars.

**Step 1** Write a function for the final cost of a kit that originally cost *c* dollars.

$f(c) = 0.80c$ *Drum kits are sold at 80% of their cost.*

## Check It Out! Example 5 Continued

**Step 2** Write a function for the final cost if the customer is a preferred customer.

$g(c) = 0.85c$ *Preferred customers receive 15% off.*

## Check It Out! Example 5 Continued

**Step 3** Find the composition $f(g(c))$.

$f(g(c)) = 0.80(g(c))$ *Substitute g(c) for c.*

$f(g(c)) = 0.80(0.85c)$ *Replace g(c) with its rule.*

$= 0.68c$

**b.** Find the cost of a drum kit at $248 that a preferred customer wants to buy.

Evaluate the composite function for $c = 248$.

$f(g(c)) = 0.68(248)$

The drum kit would cost $168.64.