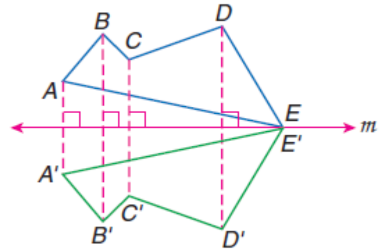
Notes 9.2 Reflections

DRAW REFLECTIONS A **reflection** is a transformation representing a flip of a figure. Figures may be reflected in a point, a line, or a plane.

The figure shows a reflection of $ABCDE$ in line m. Note that the segment connecting a point and its image is perpendicular to line m and is bisected by line m. Line m is called the **line of reflection** for $ABCDE$ and its image $A'B'C'D'E'$. Because E lies on the line of reflection, its preimage and image are the same point.

A′, A″, A‴, and so on, name corresponding points for one or more transformations.



Reflections:

$pre-image \rightarrow image$

Reflect across the x-axis: $(x, y) \rightarrow (x, -y)$

Reflect across the y-axis: $(x, y) \rightarrow (-x, y)$

Reflect across the $y = x$ line: $(x, y) \rightarrow (y, x)$

Reflect across the $y = -x$ line: $(x, y) \rightarrow (-y, -x)$

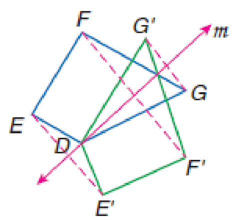
Example 1 Reflecting a Figure in a Line

Draw the reflected image of quadrilateral $DEFG$ in line m.

Step 1 Since D is on line m, D is its own reflection. Draw segments perpendicular to line m from E, F, and G.

Step 2 Locate E', F', and G' so that line m is the perpendicular bisector of $\overline{EE'}$, $\overline{FF'}$, and $\overline{GG'}$. Points E', F', and G' are the respective images of E, F, and G.

Step 3 Connect vertices D, E', F', and G'.



Since points D, E', F', and G' are the images of points D, E, F, and G under reflection in line m, then quadrilateral $DE'F'G'$ is the reflection of quadrilateral $DEFG$ in line m.

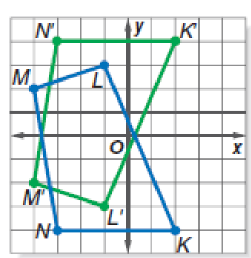
Example 2 Reflection in the x-axis

COORDINATE GEOMETRY **Quadrilateral $KLMN$ has vertices $K(2, -4)$, $L(-1, 3)$, $M(-4, 2)$, and $N(-3, -4)$. Graph $KLMN$ and its image under reflection in the x-axis. Compare the coordinates of each vertex with the coordinates of its image.**

Use the vertical grid lines to find a corresponding point for each vertex so that the x-axis is equidistant from each vertex and its image.

$K(2, -4) \rightarrow K'(2, 4)$ $L(-1, 3) \rightarrow L'(-1, -3)$

$M(-4, 2) \rightarrow M'(-4, -2)$ $N(-3, -4) \rightarrow N'(-3, 4)$



$(a, b) \rightarrow (a, -b)$

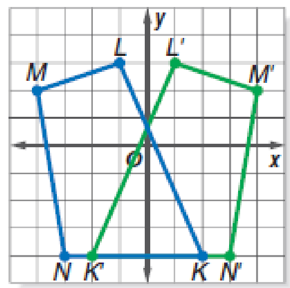
Plot the reflected vertices and connect to form the image $K'L'M'N'$. The x-coordinates stay the same, but the y-coordinates are opposites. That is, $(a, b) \rightarrow (a, -b)$.

Notes 9.2 Reflections Continued

Example 3 Reflection in the y-axis

COORDINATE GEOMETRY **Suppose quadrilateral *KLMN* from Example 2 is reflected in the *y*-axis. Graph *KLMN* and its image under reflection in the *y*-axis. Compare the coordinates of each vertex with the coordinates of its image.**

Use the horizontal grid lines to find a corresponding point for each vertex so that the *y*-axis is equidistant from each vertex and its image.



$K(2, -4) \rightarrow K'(-2, -4)$ $L(-1, 3) \rightarrow L'(1, 3)$

$M(-4, 2) \rightarrow M'(4, 2)$ $N(-3, -4) \rightarrow N'(3, -4)$

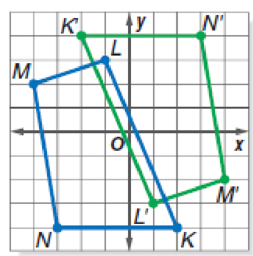
Plot the reflected vertices and connect to form the image $K'L'M'N'$. The *x*-coordinates are opposites and the *y*-coordinates are the same. That is, $(a, b) \rightarrow (-a, b)$.

$(a, b) \rightarrow (-a, b)$

Example 4 Reflection in the Origin

COORDINATE GEOMETRY **Suppose quadrilateral *KLMN* from Example 2 is reflected in the origin. Graph *KLMN* and its image under reflection in the origin. Compare the coordinates of each vertex with the coordinates of its image.**

Since $\overline{KK'}$ passes through the origin, use the horizontal and vertical distances from *K* to the origin to find the coordinates of K'. From *K* to the origin is 4 units up and 2 units left. K' is located by repeating that pattern from the origin. Four units up and 2 units left yields $K'(-2, 4)$.



$K(2, -4) \rightarrow K'(-2, 4)$ $L(-1, 3) \rightarrow L'(1, -3)$

$M(-4, 2) \rightarrow M'(4, -2)$ $N(-3, -4) \rightarrow N'(3, 4)$

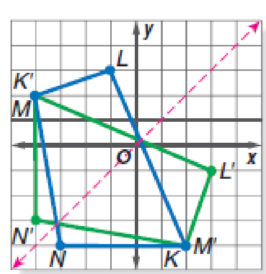
Plot the reflected vertices and connect to form the image $K'L'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (-a, -b)$.

$(a, b) \rightarrow (-a, -b)$

Example 5 Reflection in the Line y = x

COORDINATE GEOMETRY **Suppose quadrilateral *KLMN* from Example 2 is reflected in the line $y = x$. Graph *KLMN* and its image under reflection in the line $y = x$. Compare the coordinates of each vertex with the coordinates of its image.**

The slope of $y = x$ is 1. $\overline{KK'}$ is perpendicular to $y = x$, so its slope is -1. From *K* to the line $y = x$, move up three units and left three units. From the line $y = x$ move up three units and left three units to $K'(-4, 2)$.



$K(2, -4) \rightarrow K'(-4, 2)$ $L(-1, 3) \rightarrow L'(3, -1)$

$M(-4, 2) \rightarrow M'(2, -4)$ $N(-3, -4) \rightarrow N'(-4, -3)$

Plot the reflected vertices and connect to form the image $K'L'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (b, a)$.

$(a, b) \rightarrow (b, a)$