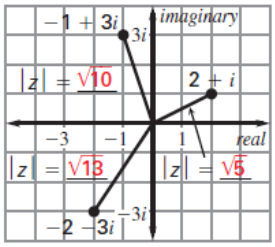
Unit 9.2 Absolute Value Equations and Complex numbers NOTES

Example 5 *Finding Absolute Values of Complex Numbers*

Find the absolute value of each complex number. Which number is farthest from the origin in the complex plane?

a. $-2 - 3i$ **b.** $2 + i$ **c.** $-1 + 3i$

Solution



a. $|-2 - 3i|$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{13} \approx 3.6$

b. $|2 + i|$

$= \sqrt{(2)^2 + (1)^2}$

$= \sqrt{5} \approx 2.2$

c. $|-1 + 3i|$

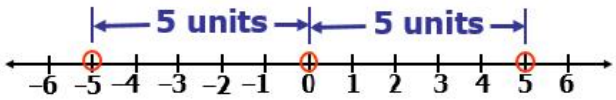
$= \sqrt{(-1)^2 + (3)^2}$

$= \sqrt{10} \approx 3.2$

Because $-2 - 3i$ has the greatest absolute value, it is farthest from the origin in the complex plane.

Recall that the absolute-value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.

For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and -5 have an absolute value of 5.

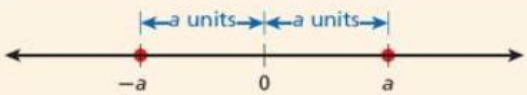


To write this statement using algebra, you would write $|x| = 5$.

This equation asks, "*What values of x have an absolute value of* 5 ?"

The solutions are 5 and -5. Notice this equation has two solutions.

Absolute-Value Equations

WORDS	NUMBERS
The equation $\|x\| = a$ asks, "What values of x have an absolute value of a?" The solutions are a and the opposite of a.	$\|x\| = 5$ $x = 5$ or $x = -5$
GRAPH	**ALGEBRA**
	$\|x\| = a$ $x = a$ or $x = -a$ $(a \geq 0)$

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation.

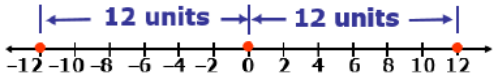
Then you must consider two cases.

Solving an Absolute-Value Equation
1. Use inverse operations to isolate the absolute-value expression.
2. Rewrite the resulting equation as two cases that do not involve absolute values.
3. Solve the equation in each of the two cases.

Solve the equation.

$|x| = 12$

$|x| = 12$ *Think: What numbers are 12 units from 0?*



Case 1	**Case 2**
$x = 12$	$x = -12$

Rewrite the equation as two cases.

The solutions are {12, −12}.

SPECIAL CASES: Not all absolute-value equations have two solutions.
If the absolute-value expression equals 0 , there is one solution.
If an equation states that an absolute-value is negative, there are no solutions.

Solve the equation.

$-8 = |x + 2| - 8$

$-8 = |x + 2| - 8$
$+8 \quad\quad + 8$
$0 = |x + 2|$

Since 8 is subtracted from $|x + 2|$, add 8 to both sides to undo the subtraction.

$0 = x + 2$
$-2 \quad\quad -2$
$-2 = x$

There is only one case. Since 2 is added to x, subtract 2 from both sides to undo the addition.

The solution is {−2}.

Solve the equation.

$3 + |x + 4| = 0$

$3 + |x + 4| = 0$
$-3 \quad\quad -3$
$|x + 4| = -3$ ×

Since 3 is added to $|x + 4|$, subtract 3 from both sides to undo the addition.

Absolute value cannot be negative.

This equation has no solution.