

Unit 8.5 Finding Zeros of a Polynomial Advanced PRACTICE

Period _____

State the possible rational zeros for each function. Then factor each and find all zeros.

1) $f(x) = 5x^5 + 15x^4 - 4x^3 - 12x^2 - 9x - 27$

Possible rational zeros:

$$\pm 1, \pm 3, \pm 9, \pm 27, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}, \pm \frac{27}{5}$$

Factors to: $f(x) = (x + 3)(5x^2 - 9)(x^2 + 1)$

Zeros: $\left\{-3, \frac{3\sqrt{5}}{5}, -\frac{3\sqrt{5}}{5}, i, -i\right\}$

2) $f(x) = 3x^5 - 15x^4 + 5x^3 - 25x^2 + 2x - 10$

Possible rational zeros:

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$$

Factors to: $f(x) = (x - 5)(3x^2 + 2)(x^2 + 1)$

Zeros: $\left\{5, \frac{i\sqrt{6}}{3}, -\frac{i\sqrt{6}}{3}, i, -i\right\}$

3) $f(x) = 3x^5 + 15x^4 - 4x^3 - 20x^2 - 15x - 75$

Possible rational zeros:

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm 25, \pm 75, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{25}{3}$$

Factors to: $f(x) = (x + 5)(x^2 - 3)(3x^2 + 5)$

Zeros: $\left\{-5, \sqrt{3}, -\sqrt{3}, \frac{i\sqrt{15}}{3}, -\frac{i\sqrt{15}}{3}\right\}$

4) $f(x) = 5x^4 - 3x^2 - 14$

Possible rational zeros:

$$\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{7}{5}, \pm \frac{14}{5}$$

Factors to: $f(x) = (5x^2 + 7)(x^2 - 2)$

Zeros: $\left\{\frac{i\sqrt{35}}{5}, -\frac{i\sqrt{35}}{5}, \sqrt{2}, -\sqrt{2}\right\}$

5) $f(x) = 25x^5 + 5x^4 + 220x^3 + 44x^2 - 45x - 9$

Possible rational zeros:

$$\pm 1, \pm 3, \pm 9, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}, \pm \frac{1}{25}, \pm \frac{3}{25}, \pm \frac{9}{25}$$

Factors to: $f(x) = (5x + 1)(5x^2 - 1)(x^2 + 9)$

Zeros: $\left\{-\frac{1}{5}, \frac{\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}, 3i, -3i\right\}$

6) $f(x) = 15x^5 - 5x^4 - 48x^3 + 16x^2 + 9x - 3$

Possible rational zeros:

$$\pm 1, \pm 3, \pm \frac{1}{3}, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{1}{15}$$

Factors to: $f(x) = (3x - 1)(x^2 - 3)(5x^2 - 1)$

Zeros: $\left\{\frac{1}{3}, \sqrt{3}, -\sqrt{3}, \frac{\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right\}$

$$7) f(x) = 3x^4 + 8x^2 + 5$$

Possible rational zeros: $\pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3}$

Factors to: $f(x) = (3x^2 + 5)(x^2 + 1)$

$$\text{Zeros: } \left\{ \frac{i\sqrt{15}}{3}, -\frac{i\sqrt{15}}{3}, i, -i \right\}$$

$$8) f(x) = 2x^4 + 3x^2 - 27$$

Possible rational zeros:

$$\pm 1, \pm 3, \pm 9, \pm 27, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{27}{2}$$

Factors to: $f(x) = (x^2 - 3)(2x^2 + 9)$

$$\text{Zeros: } \left\{ \sqrt{3}, -\sqrt{3}, \frac{3i\sqrt{2}}{2}, -\frac{3i\sqrt{2}}{2} \right\}$$

$$9) f(x) = 6x^5 - 3x^4 - 8x^3 + 4x^2 - 8x + 4$$

Possible rational zeros:

$$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$$

Factors to: $f(x) = (2x - 1)(x^2 - 2)(3x^2 + 2)$

$$\text{Zeros: } \left\{ \frac{1}{2}, \sqrt{2}, -\sqrt{2}, \frac{i\sqrt{6}}{3}, -\frac{i\sqrt{6}}{3} \right\}$$

$$10) f(x) = 5x^4 + 23x^2 - 10$$

Possible rational zeros:

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{5}, \pm \frac{2}{5}$$

Factors to: $f(x) = (5x^2 - 2)(x^2 + 5)$

$$\text{Zeros: } \left\{ \frac{\sqrt{10}}{5}, -\frac{\sqrt{10}}{5}, i\sqrt{5}, -i\sqrt{5} \right\}$$

$$11) f(x) = 2x^5 - 10x^4 + 11x^3 - 55x^2 + 5x - 25$$

Possible rational zeros:

$$\pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}$$

Factors to: $f(x) = (x - 5)(x^2 + 5)(2x^2 + 1)$

$$\text{Zeros: } \left\{ 5, i\sqrt{5}, -i\sqrt{5}, \frac{i\sqrt{2}}{2}, -\frac{i\sqrt{2}}{2} \right\}$$

$$12) f(x) = 2x^4 - 9x^2 - 35$$

Possible rational zeros:

$$\pm 1, \pm 5, \pm 7, \pm 35, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{35}{2}$$

Factors to: $f(x) = (2x^2 + 5)(x^2 - 7)$

$$\text{Zeros: } \left\{ \frac{i\sqrt{10}}{2}, -\frac{i\sqrt{10}}{2}, \sqrt{7}, -\sqrt{7} \right\}$$

$$13) f(x) = 10x^5 - 5x^4 + 28x^3 - 14x^2 - 6x + 3$$

Possible rational zeros:

$$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{1}{10}, \pm \frac{3}{10}$$

Factors to: $f(x) = (2x - 1)(5x^2 - 1)(x^2 + 3)$

$$\text{Zeros: } \left\{ \frac{1}{2}, \frac{\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}, i\sqrt{3}, -i\sqrt{3} \right\}$$

$$14) f(x) = 3x^5 - 15x^4 + 2x^3 - 10x^2 - x + 5$$

Possible rational zeros: $\pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3}$

Factors to: $f(x) = (x - 5)(x^2 + 1)(3x^2 - 1)$

$$\text{Zeros: } \left\{ 5, i, -i, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3} \right\}$$