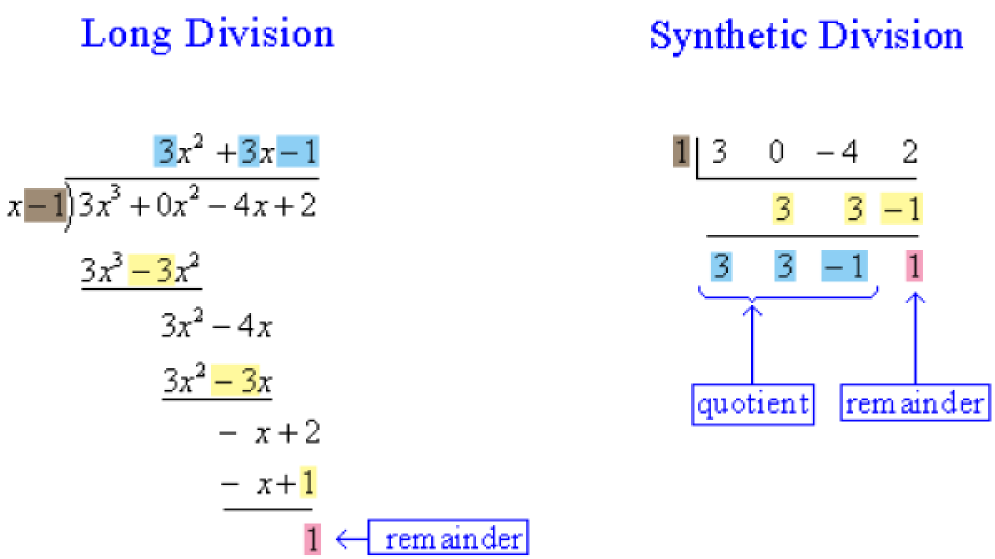
Unit 8.3 Synthetic Division of Polynomials NOTES

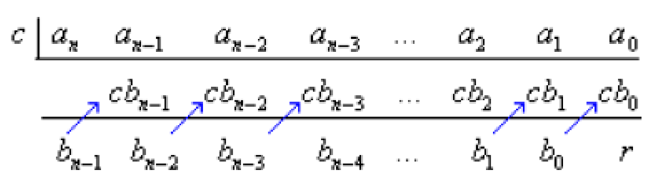
Synthetic Division:

Synthetic division is a shortcut method of performing long division that can be used when the divisor is a first degree polynomial of the form $x - c$. In synthetic division we write only the essential part of the long division table. To illustrate, compare these long division and synthetic division tables, in which we divide $3x^3 - 4x + 2$ by $x - 1$:



Note that in synthetic division we abbreviate $3x^3 - 4x + 2$ by writing only the coefficients: 3 0 –4 2, and instead of $x - 1$, we simply write 1. (Writing 1 instead of –1 allows us to add instead of subtract, but this changes the sign of all the numbers that appear in the yellow boxes.)

To divide $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ by $x - c$, we proceed as follows:



Here $b_{n-1} = a_n$, and each number in the bottom row is obtained by adding the numbers above it. The remainder is r and the quotient is

$$b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \ldots + b_1x + b_0$$

Example 4: Find the quotient and the remainder of $\dfrac{x^4 - 7x^2 - 6x}{x+2}$ using synthetic division.

Solution:

Step 1: We put $x + 2$ in the form $x - c$ by writing it as $x - (-2)$. Use this and the coefficients of the polynomial to obtain

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline \end{array}$$

Note that we used 0 as the coefficient of any missing powers.

Step 2: Next, bring down the 1.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline & & & & & \\ \hline & 1 & & & & \end{array}$$

Step 3: Now, multiply -2 by 1 to get -2, and add it to the 0 in the first row. The result is -2.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline & & -2 & & & \\ \hline & 1 & -2 & & & \end{array}$$

Step 4: Next, $-2(-2) = 4$. Add this to the -7 in the first row.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline & & -2 & 4 & & \\ \hline & 1 & -2 & -3 & & \end{array}$$

Example 4 (Continued):

Step 5: $-2(-3) = 6$. Add this to the -6 in the first row.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline & & -2 & 4 & 6 & \\ \hline & 1 & -2 & -3 & 0 & \end{array}$$

Step 6: Finally, $-2(0) = 0$, which is added to 0 to get 0.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -7 & -6 & 0 \\ \hline & & -2 & 4 & 6 & 0 \\ \hline & 1 & -2 & -3 & 0 & 0 \end{array}$$

The coefficients of the quotient polynomial and the remainder are read directly from the bottom row. Also, the degree of the quotient will always be one less than the degree of the dividend. Thus, $Q(x) = x^3 - 2x^2 - 3x$ and $R(x) = 0$.

The Remainder and Factor Theorems:

Synthetic division can be used to find the values of polynomials in a sometimes easier way than substitution. This is shown by the next theorem.

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Example 5: Use synthetic division and the Remainder Theorem to evaluate $P(c)$ if $P(x) = x^3 - 4x^2 + 2x - 1$, $c = -1$.

Solution:

Step1: First we will use synthetic division to divide $P(x) = x^3 - 4x^2 + 2x - 1$ by $x - (-1)$.

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 2 & -1 \\ \hline & & -1 & 5 & -7 \\ \hline & 1 & -5 & 7 & -8 \end{array}$$

Step 2: Since the remainder when $P(x)$ is divided by $x - (-1) = x + 1$ is -8, by the Remainder Theorem, $P(-1) = -8$.

We learned that if c is a *zero* of P, than $x - c$ is a *factor* of $P(x)$. The next theorem restates this fact in a more useful way.

Factor Theorem: c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Example 6: Use the Factor Theorem to show that $x+\frac{1}{2}$ is a factor of $P(x) = 2x^3 + 5x^2 + 4x + 1$.

Solution:

In order to show that $x+\frac{1}{2}$ is a factor of $P(x) = 2x^3 + 5x^2 + 4x + 1$, we must show that $-\frac{1}{2}$ is a zero of P, or that $P\left(-\frac{1}{2}\right)=0$. We will use synthetic division and the Remainder Theorem to do this.

Step 1: Use synthetic division to divide $P(x) = 2x^3 + 5x^2 + 4x + 1$ by $x-\left(-\frac{1}{2}\right)$.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & 4 & 1 \\ & & -1 & -2 & -1 \\ \hline & 2 & 4 & 2 & 0 \end{array}$$

Step 2: Since the remainder is 0, by the Remainder Theorem, we know $P\left(-\frac{1}{2}\right)=0$.

Step 3: Finally, since $P\left(-\frac{1}{2}\right)=0$, we know that $-\frac{1}{2}$ is a zero of P, by definition. Hence, by the Factor Theorem, $x+\frac{1}{2}$ is a factor of $P(x) = 2x^3 + 5x^2 + 4x + 1$.