

## Unit 7 Logarithms Notes

Logarithms or “logs” for short is used when a variable is in the exponent.

Example:  $5^x = 6$

This can only be solved by using logarithms.

### History of logarithms

Before computers and calculators ancient mathematicians worked out the problems by hand and created logarithm tables. Whole books were made of just logarithm tables. Then instead of doing the complicated calculations by hand the mathematician would use the logarithm tables to find the correct calculations thereby speeding up the process. Leonhard Euler discovered a new number that is used often with logarithms in the natural sciences. It became as famous as “ $\pi$ ” and “ $i$ ”. The number was denoted by the letter “ $e$ ” and is called the natural base  $e$  or the Euler number. “ $e$ ” is approximately 2.718281828459...

### Rules:

#### Definition of logarithm:

Any equation in form:  $a^x = y$  can be rewritten as:  $\log_a y = x$  (read as “log base  $a$  of  $y$  equals  $x$ ”)

Or  
 $e^x = y$  can be rewritten as:  $\ln y = x$  (read as “natural log of  $y$  equals  $x$ ”)

Log base 10 is called the “common logarithm” and is written as:  $\log y = x$  which shows no log base

Log base  $e$  is called the “natural logarithm” and is written as:  $\ln$  which would be the same as:  $\log_e$

**Special logarithms:**  $\log_a 1 = 0$  because  $a^0 = 1$

and  $\log_a a = 1$  because  $a^1 = a$

and log base 1 does not exist because  $1^x$  cannot equal anything other than 1.

**Product Property:**  $\log_a xy = \log_a x + \log_a y$

**Quotient Property:**  $\log_a \frac{x}{y} = \log_a x - \log_a y$

**Power Property:**  $\log_a x^y = y \log_a x$

#### Change of base formula:

$$\log_a x = \frac{\log x}{\log a}$$