## SIMILAR TRIANGLES

Objectives:

After completing this section, you should be able to do the following:

- Calculate the lengths of sides of similar triangles.
- Solve word problems involving similar triangles.

Vocabulary:

As you read, you should be looking for the following vocabulary words and their definitions:

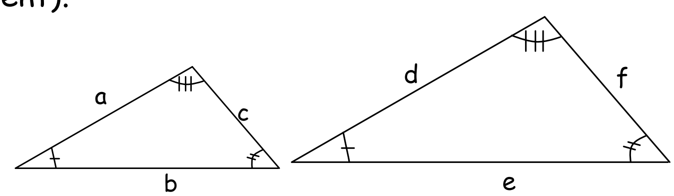
- similar triangles

Formulas:

You should be looking for the following formulas as you read:

- proportions for similar triangles

We will continue our study of geometry by looking at similar triangles. Two triangles are *similar* is their corresponding angles are congruent (or have the same measurement). similar




In the picture above, the corresponding angles are indicated in the two triangles by the same number of hash marks. In other words, the angle with one hash mark in the smaller triangle corresponds to the angle with one hash mark in the larger triangle. These angles have the same measure or are congruent.

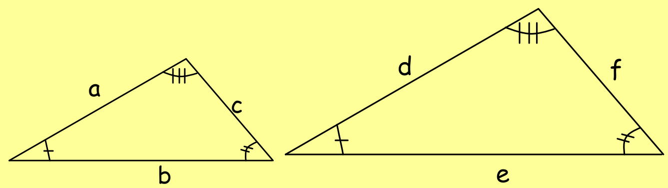
There are also corresponding sides in similar triangles. In the triangles above side $a$ corresponds to side $d$ (for example). These sides do not necessarily have the same measure. They do, however, form a ratio that is the same no matter which pair of corresponding sides the ratio is made from. Thus we can write the following equation

$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

Notice that the sides of one particular triangle are always written on top of the fractions and the sides of the other triangle are always written on the bottom of the fractions. It does not matter which triangle is put in which part of the fraction as longs as we are consistent within a problem.

Similar Triangle Ratios

$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

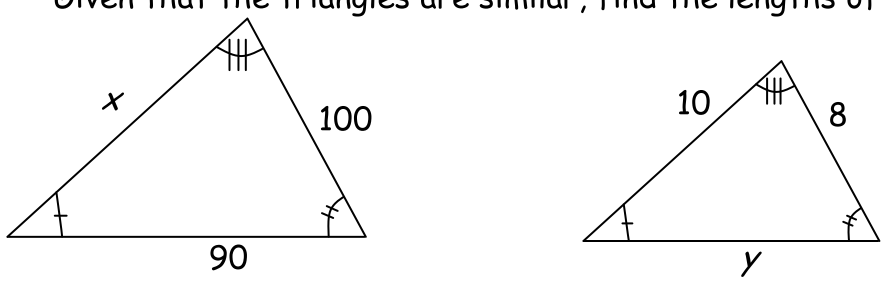



It should be noted that although our triangles are in the same relative position, this is not needed for triangles to be similar. One of the triangles can be rotated or reflected.

Please note that pictures below are not drawn to scale.

Example 1:

Given that the triangles are similar, find the lengths of the missing sides.




Solution:

There is one side missing in the triangle on the left. This side is labeled $x$. There is also one side missing from the triangle on the right. This side is labeled $y$. The side $x$ in the triangle on the left corresponds to the side labeled 10 in the triangle on the right. We know this because these sides connect the angle with one hash mark to the angle with three hash marks in each of the triangles. The side 100 in the triangle on the left corresponds to the side labeled 8 in the triangle on the right. Finally the side 90 in the triangle on the left corresponds to the side labeled $y$ in the triangle on the right. We will need this information to find values for $x$ and $y$.

Find $x$:
We will start by finding the value for $x$. We will need to form a ratio with the side labeled $x$ and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just need to solve the equation for $x$. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

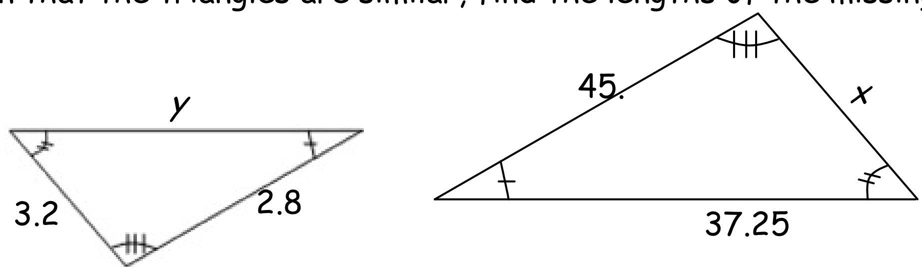
$$\frac{x}{10} = \frac{100}{8}$$
$$8(x) = 10(100)$$
$$8x = 1000$$
$$x = 125 \text{ units}$$

Find $y$:
We will continue by finding the value for $y$. We will need to form a ratio with the side labeled $y$ and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Although we know the measurements of all the other sides of the triangle, it is best to avoid using a side we just calculate to make further calculations if possible. The reason for this is that if we have made a mistake in our previous calculation, we would end up with an incorrect answer here as well. Once we make an equation using these ratios, we will just need to solve the equation for $y$. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{90}{y} = \frac{100}{8}$$
$$8(90) = y(100)$$
$$720 = 100y$$
$$7.2 \text{ units} = y$$

## Example 2:

Given that the triangles are similar, find the lengths of the missing sides.




Solution:

Our first job here is to determine which sides in the triangle on the left correspond to which sides in the triangle on the right. The side in the triangle on the left labeled $y$ joins the angle with one hash mark and the angle with two has marks. The side in the triangle on the right that does this is the side labeled 37.25. Thus these are corresponding sides.

In a similar manner, we can see that the side labeled 3.2 in the triangle on the left corresponds to the side labeled $x$ on the triangle on the right. Finally the side labeled 2.8 in the triangle on the left corresponds to the side labeled 45 in the triangle on the right.

Find $x$:
We will start by finding the value for $x$. We will need to form a ratio with the side labeled $x$ and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just

need to solve the equation for $x$. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{3.2}{x} = \frac{2.8}{45}$$
$$3.2(45) = x(2.8)$$
$$144 = 2.8x$$
$$\frac{144}{2.8}\text{ units } = x$$

This is approximately 51.42857 units.

Find $y$:
We will continue by finding the value for $y$. We will need to form a ratio with the side labeled $y$ and is corresponding side in the triangle on the right. We will also need a pair of corresponding side both of whose measurements we have and make a ratio out of those. Once we make an equation using these ratios, we will just need to solve the equation for $y$. I am going to choose to place the sides of the triangle on the left on the top of each of the ratios.

$$\frac{y}{37.25} = \frac{2.8}{45}$$
$$y(45) = 37.25(2.8)$$
$$45y = 104.3$$
$$y = \frac{104.3}{45}\text{ units}$$

This is approximately 3.21778 units.

Our final example is an application of similar triangles

Example 3:

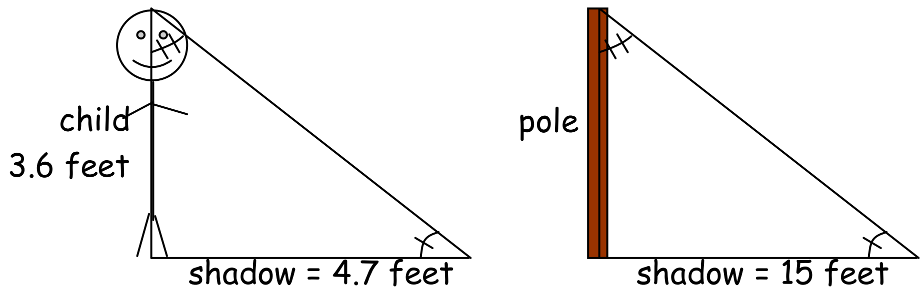
A 3.6-foot –tall child casts a shadow of 4.7 feet at the same instant that a telephone pole casts a shadow of 15 feet. How tall is the telephone pole?

Solution:

For this problem, it helps to have a picture.

# Geometry Notes






When we look at the picture above, we see that we have two triangles that are similar. We are looking for the height of the pole. The pole side of the triangle on the right corresponds to the side of the child side of the triangle on the left. The two shadow sides of the triangles also correspond to each other. Thus we can write an equation of ratios to find the height of the pole.

$$\frac{3.6}{\text{pole}} = \frac{4.7}{15}$$

$$(3.6)(15) = \text{pole}(4.7)$$

$$54 = 4.7\text{pole}$$

$$\frac{54}{4.7} = \text{pole}$$

Thus the pole is approximately 11.48936 feet high.