Math 3 Name_____

Unit 4.2 Fundamental Identities Advanced PRACTICE

Period____

Verify each identity.

1) $\dfrac{\csc^2 x}{\sec^3 x} = \dfrac{\cos x}{\sec^2 x - 1}$

$\dfrac{\csc^2 x}{\sec^3 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\csc^2 x \cos^3 x$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\cos^3 x}{\sin^2 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\cos x}{\tan^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\cos x}{\sec^2 x - 1}$ ■

2) $\dfrac{\sin^2 x}{\cos^2 x \cot x} = \tan x \cdot \left(\sec^2 x - 1\right)$

$\dfrac{\sin^2 x}{\cos^2 x \cot x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\tan^2 x}{\cot x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\sec^2 x - 1}{\cot x}$ Use $\cot x = \dfrac{1}{\tan x}$

$\tan x \cdot \left(\sec^2 x - 1\right)$ ■

3) $\dfrac{\sin x - \tan x}{\tan x} = \tan^2 x + \cos x - \sec^2 x$

$\dfrac{\sin x - \tan x}{\tan x}$ Decompose into sine and cosine

$\dfrac{\sin x - \dfrac{\sin x}{\cos x}}{\dfrac{\sin x}{\cos x}}$ Simplify

$\cos x - 1$ Use $\tan^2 x + 1 = \sec^2 x$

$\tan^2 x + \cos x - \sec^2 x$ ■

4) $\dfrac{\tan^2 x + 1}{\sin^2 x} = \dfrac{\csc^2 x}{\cos^2 x}$

$\dfrac{\tan^2 x + 1}{\sin^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\sec^2 x}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc^2 x \sec^2 x$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\csc^2 x}{\cos^2 x}$ ■

5) $\dfrac{\cot x}{\sec^2 x + \csc^2 x} = \cos^3 x \sin x$

$\dfrac{\cot x}{\sec^2 x + \csc^2 x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\cos x}{\sin x}}{\left(\dfrac{1}{\cos x}\right)^2 + \left(\dfrac{1}{\sin x}\right)^2}$ Simplify

$\dfrac{\cos^3 x \sin x}{\sin^2 x + \cos^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\cos^3 x \sin x$ ■

6) $\cot^2 x \csc^2 x \sin^2 x = \csc^2 x - 1$

$\cot^2 x \csc^2 x \sin^2 x$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\cot^2 x \sin^2 x}{\sin^2 x}$ Cancel common factors

$\cot^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\csc^2 x - 1$ ■

7) $\dfrac{\cot^2 x}{\csc^2 x + \sec^2 x} = \dfrac{\cos^3 x}{\sec x}$

$\dfrac{\cot^2 x}{\csc^2 x + \sec^2 x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{\cos x}{\sin x}\right)^2}{\left(\dfrac{1}{\sin x}\right)^2 + \left(\dfrac{1}{\cos x}\right)^2}$ Simplify

$\dfrac{\cos^4 x}{\cos^2 x + \sin^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\cos^4 x$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\cos^3 x}{\sec x}$ ■

8) $\dfrac{\cot^2 x - \csc^2 x}{\cot x} = -\sec x \sin x$

$\dfrac{\cot^2 x - \csc^2 x}{\cot x}$ Use $\cot^2 x + 1 = \csc^2 x$

$-\dfrac{1}{\cot x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$-\dfrac{\sin x}{\cos x}$ Use $\sec x = \dfrac{1}{\cos x}$

$-\sec x \sin x$ ■

9) $-\tan^2 x \csc^2 x = -\tan^2 x - 1$

$-\tan^2 x \csc^2 x$ Decompose into sine and cosine

$-\left(\dfrac{\sin x}{\cos x}\right)^2 \cdot \left(\dfrac{1}{\sin x}\right)^2$ Simplify

$-\dfrac{1}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$-\sec^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$-\tan^2 x - 1$ ■

10) $\dfrac{\cot^2 x \sec^2 x}{\cos^2 x} = \dfrac{\tan^2 x + 1}{\sin^2 x}$

$\dfrac{\cot^2 x \sec^2 x}{\cos^2 x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\dfrac{\sec^2 x \cos^2 x}{\cos^2 x \sin^2 x}$ Cancel common factors

$\dfrac{\sec^2 x}{\sin^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan^2 x + 1}{\sin^2 x}$ ■

11) $\csc^2 x \cot x \tan x = \cot^2 x + 1$

$\csc^2 x \cot x \tan x$ Use $\cot x = \dfrac{1}{\tan x}$

$\dfrac{\csc^2 x \tan x}{\tan x}$ Cancel common factors

$\csc^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\cot^2 x + 1$ ■

12) $\csc^2 x - 1 = \dfrac{\cot x \csc x}{\sec x}$

$\csc^2 x - 1$ Use $\cot^2 x + 1 = \csc^2 x$

$\cot^2 x$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\dfrac{\cot x \cos x}{\sin x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\cot x \csc x \cos x$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\cot x \csc x}{\sec x}$ ■

13) $\dfrac{\csc^2 x - 1}{\csc x} = \csc x \cos^2 x$

$\dfrac{\csc^2 x - 1}{\csc x}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\cot^2 x}{\csc x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{\cos x}{\sin x}\right)^2}{\dfrac{1}{\sin x}}$ Simplify

$\dfrac{\cos^2 x}{\sin x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc x \cos^2 x$ ■

14) $\dfrac{\cos^2 x}{1 - \csc^2 x} = -\sin^2 x$

$\dfrac{\cos^2 x}{1 - \csc^2 x}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\cos^2 x}{-\cot^2 x}$ Decompose into sine and cosine

$\dfrac{\cos^2 x}{-\left(\dfrac{\cos x}{\sin x}\right)^2}$ Simplify

$-\sin^2 x$ ■

15) $-\csc x \sin^2 x = \dfrac{\tan^2 x - \sec^2 x}{\csc x}$

$-\csc x \sin^2 x$ Use $\csc x = \dfrac{1}{\sin x}$

$-\dfrac{\csc x}{\csc^2 x}$ Cancel common factors

$-\dfrac{1}{\csc x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan^2 x - \sec^2 x}{\csc x}$ ■

16) $\dfrac{1}{\cos^2 x \csc^2 x} = \sec^2 x - 1$

$\dfrac{1}{\cos^2 x \csc^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\sin^2 x}{\cos^2 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 1$ ■

17) $\dfrac{\csc x}{\csc^2 x - 1} = \sin x \sec^2 x$

$\dfrac{\csc x}{\csc^2 x - 1}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\csc x}{\cot^2 x}$ Decompose into sine and cosine

$\dfrac{\dfrac{1}{\sin x}}{\left(\dfrac{\cos x}{\sin x}\right)^2}$ Simplify

$\dfrac{\sin x}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\sin x \sec^2 x$ ■

18) $\cot^2 x + \sec x \cos x = \dfrac{1}{\sin^2 x}$

$\cot^2 x + \sec x \cos x$ Decompose into sine and cosine

$\left(\dfrac{\cos x}{\sin x}\right)^2 + \dfrac{1}{\cos x} \cdot \cos x$ Simplify

$\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\sin^2 x}$ ■