Math 3   Name_____
Unit 4.2 Fundamental Identities Advanced EXAMPLES

Verify each identity.

1) $\dfrac{\tan^2 x + 1}{\tan^2 x} = \dfrac{\csc x}{\sin x}$

2) $\sec^2 x + \csc^2 x = \dfrac{\csc^2 x}{\cos^2 x}$

3) $\dfrac{\tan x + \cot x}{\cot^2 x} = \dfrac{\tan x}{\cos^2 x}$

4) $\dfrac{\tan x}{\sec^2 x - 1} = \dfrac{\cos x}{\sin x}$

5) $\sin^2 x \csc^2 x \cot^2 x = \csc^2 x - 1$

6) $\dfrac{\tan x}{1 - \sec^2 x} = -\cot x$

7) $\dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$

8) $\cot x \tan^3 x = \dfrac{\sin^2 x + \cos^2 x}{\cot^2 x}$

9) $\dfrac{\tan^2 x}{\cot x \sec x} = \sin x \cdot (\sec^2 x - 1)$

10) $\cot x \cdot (\tan x + \cot x) = \dfrac{1}{\sin^2 x}$

11) $\csc^2 x \tan^2 x = \dfrac{\sec^2 x}{\cos^2 x + \sin^2 x}$

12) $\sec^2 x \left(1 - \csc^2 x\right) = -\csc^2 x$

13) $\tan^2 x \csc x = \sin x \cdot \left(\tan^2 x + 1\right)$

14) $\dfrac{\tan x + \cot x}{\sec^2 x} = \csc x \cos x$

15) $\dfrac{\cot^2 x + 1}{\cot^2 x} = \dfrac{\sec x}{\cos x}$

16) $\dfrac{1}{\cot x + \tan x} = \cos x \sin x$

17) $\cos x \tan^2 x \sec x = \sec^2 x - 1$

18) $\dfrac{\cos^2 x}{\csc^2 x \cot^2 x} = \dfrac{\sin^2 x}{1 + \cot^2 x}$

Unit 4.2 Fundamental Identities Advanced EXAMPLES

Verify each identity.

1) $\dfrac{\tan^2 x + 1}{\tan^2 x} = \dfrac{\csc x}{\sin x}$

$\dfrac{\tan^2 x + 1}{\tan^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\sec^2 x}{\tan^2 x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{1}{\cos x}\right)^2}{\left(\dfrac{\sin x}{\cos x}\right)^2}$ Simplify

$\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\csc x}{\sin x}$ ■

2) $\sec^2 x + \csc^2 x = \dfrac{\csc^2 x}{\cos^2 x}$

$\sec^2 x + \csc^2 x$ Decompose into sine and cosine

$\left(\dfrac{1}{\cos x}\right)^2 + \left(\dfrac{1}{\sin x}\right)^2$ Simplify

$\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\sin^2 x \cos^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\csc^2 x}{\cos^2 x}$ ■

3) $\dfrac{\tan x + \cot x}{\cot^2 x} = \dfrac{\tan x}{\cos^2 x}$

$\dfrac{\tan x + \cot x}{\cot^2 x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}{\left(\dfrac{\cos x}{\sin x}\right)^2}$ Simplify

$\dfrac{\sin x \cdot \left(\sin^2 x + \cos^2 x\right)}{\cos^3 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{\sin x}{\cos^3 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\tan x}{\cos^2 x}$ ■

4) $\dfrac{\tan x}{\sec^2 x - 1} = \dfrac{\cos x}{\sin x}$

$\dfrac{\tan x}{\sec^2 x - 1}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan x}{\tan^2 x}$ Cancel common factors

$\dfrac{1}{\tan x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\cos x}{\sin x}$ ■

5) $\sin^2 x\csc^2 x\cot^2 x = \csc^2 x - 1$

$\sin^2 x\csc^2 x\cot^2 x$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\csc^2 x\cot^2 x}{\csc^2 x}$ Cancel common factors

$\cot^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\csc^2 x - 1$ ■

6) $\dfrac{\tan x}{1 - \sec^2 x} = -\cot x$

$\dfrac{\tan x}{1 - \sec^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan x}{-\tan^2 x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\sin x}{\cos x}}{-\left(\dfrac{\sin x}{\cos x}\right)^2}$ Simplify

$-\dfrac{\cos x}{\sin x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$-\cot x$ ■

7) $\dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$\dfrac{\tan^2 x}{\sin^2 x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{\sin x}{\cos x}\right)^2}{\sin^2 x}$ Simplify

$\dfrac{1}{\cos^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$ ■

8) $\cot x \tan^3 x = \dfrac{\sin^2 x + \cos^2 x}{\cot^2 x}$

$\cot x \tan^3 x$ Use $\cot x = \dfrac{1}{\tan x}$

$\dfrac{\cot x \tan^2 x}{\cot x}$ Cancel common factors

$\tan^2 x$ Use $\sin^2 x + \cos^2 x = 1$

$\tan^2 x\left(\sin^2 x + \cos^2 x\right)$ Use $\cot x = \dfrac{1}{\tan x}$

$\dfrac{\sin^2 x + \cos^2 x}{\cot^2 x}$ ■

9) $\dfrac{\tan^2 x}{\cot x \sec x} = \sin x \cdot \left(\sec^2 x - 1\right)$

$\dfrac{\tan^2 x}{\cot x \sec x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\dfrac{\tan^2 x \sin x}{\sec x \cos x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\tan^2 x \cos x \sin x}{\cos x}$ Cancel common factors

$\sin x \tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sin x \cdot \left(\sec^2 x - 1\right)$ ∎

10) $\cot x \cdot \left(\tan x + \cot x\right) = \dfrac{1}{\sin^2 x}$

$\cot x \cdot \left(\tan x + \cot x\right)$ Decompose into sine and cosine

$\dfrac{\cos x}{\sin x}\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$ Simplify

$\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\sin^2 x}$ ∎

11) $\csc^2 x \tan^2 x = \dfrac{\sec^2 x}{\cos^2 x + \sin^2 x}$

$\csc^2 x \tan^2 x$ Decompose into sine and cosine

$\left(\dfrac{1}{\sin x}\right)^2 \cdot \left(\dfrac{\sin x}{\cos x}\right)^2$ Simplify

$\dfrac{1}{\cos^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\cos^2 x \left(\cos^2 x + \sin^2 x\right)}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sec^2 x}{\cos^2 x + \sin^2 x}$ ∎

12) $\sec^2 x \left(1 - \csc^2 x\right) = -\csc^2 x$

$\sec^2 x \left(1 - \csc^2 x\right)$ Use $\cot^2 x + 1 = \csc^2 x$

$-\sec^2 x \cot^2 x$ Decompose into sine and cosine

$-\left(\dfrac{1}{\cos x}\right)^2 \cdot \left(\dfrac{\cos x}{\sin x}\right)^2$ Simplify

$-\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$-\csc^2 x$ ∎

13) $\tan^2 x \csc x = \sin x \cdot \left(\tan^2 x + 1\right)$

$\tan^2 x \csc x$ Decompose into sine and cosine

$\left(\dfrac{\sin x}{\cos x}\right)^2 \cdot \dfrac{1}{\sin x}$ Simplify

$\dfrac{\sin x}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\sin x \sec^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sin x \cdot \left(\tan^2 x + 1\right)$ ∎

14) $\dfrac{\tan x + \cot x}{\sec^2 x} = \csc x \cos x$

$\dfrac{\tan x + \cot x}{\sec^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\tan x + \cot x}{\tan^2 x + 1}$ Decompose into sine and cosine

$\dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}{\left(\dfrac{\sin x}{\cos x}\right)^2 + 1}$ Simplify

$\dfrac{\cos x}{\sin x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc x \cos x$ ∎

15) $\dfrac{\cot^2 x + 1}{\cot^2 x} = \dfrac{\sec x}{\cos x}$

$\dfrac{\cot^2 x + 1}{\cot^2 x}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\csc^2 x}{\cot^2 x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{1}{\sin x}\right)^2}{\left(\dfrac{\cos x}{\sin x}\right)^2}$ Simplify

$\dfrac{1}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sec x}{\cos x}$ ■

16) $\dfrac{1}{\cot x + \tan x} = \cos x \sin x$

$\dfrac{1}{\cot x + \tan x}$ Decompose into sine and cosine

$\dfrac{1}{\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}}$ Simplify

$\dfrac{\sin x \cos x}{\cos^2 x + \sin^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\cos x \sin x$ ■

17) $\cos x \tan^2 x \sec x = \sec^2 x - 1$

$\cos x \tan^2 x \sec x$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\cos x \tan^2 x}{\cos x}$ Cancel common factors

$\tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 1$ ∎

18) $\dfrac{\cos^2 x}{\csc^2 x \cot^2 x} = \dfrac{\sin^2 x}{1 + \cot^2 x}$

$\dfrac{\cos^2 x}{\csc^2 x \cot^2 x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\dfrac{\cos^2 x \sin^2 x}{\csc^2 x \cos^2 x}$ Cancel common factors

$\dfrac{\sin^2 x}{\csc^2 x}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\sin^2 x}{1 + \cot^2 x}$ ∎