

Unit 4.1 Fundamental Identities Basic PRACTICE

Verify each identity.

$$1) \cot^2 x + \csc^2 x = \frac{1 + \cos^2 x}{\sin^2 x}$$

$$\cot^2 x + \csc^2 x \quad \text{Decompose into sine and cosine}$$

$$\left(\frac{\cos x}{\sin x}\right)^2 + \left(\frac{1}{\sin x}\right)^2 \quad \text{Simplify}$$

$$\frac{1 + \cos^2 x}{\sin^2 x} \quad \blacksquare$$

$$2) \frac{\tan^2 x}{\sin x \sec^2 x} = \sin x$$

$$\frac{\tan^2 x}{\sin x \sec^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\left(\frac{\sin x}{\cos x}\right)^2}{\sin x \cdot \left(\frac{1}{\cos x}\right)^2} \quad \text{Simplify}$$

$$\sin x \quad \blacksquare$$

$$3) \frac{1}{1 + \csc x} = \frac{\sin x}{\sin x + 1}$$

$$\frac{1}{1 + \csc x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{1 + \frac{1}{\sin x}} \quad \text{Simplify}$$

$$\frac{\sin x}{\sin x + 1} \quad \blacksquare$$

$$4) \frac{\cot x}{\cot x + 1} = \frac{\cos x}{\sin x + \cos x}$$

$$\frac{\cot x}{\cot x + 1} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x} + 1} \quad \text{Simplify}$$

$$\frac{\cos x}{\sin x + \cos x} \quad \blacksquare$$

$$5) \frac{\sec x}{\tan x + \sec x} = \frac{1}{1 + \sin x}$$

$$\frac{\sec x}{\tan x + \sec x}$$

Decompose into sine and cosine

$$\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x} + \frac{1}{\cos x}}$$

Simplify

$$\frac{1}{1 + \sin x}$$

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$$6) \frac{\cos x}{\sec x \cot^2 x} = \sin^2 x$$

$$\frac{\cos x}{\sec x \cot^2 x}$$

Decompose into sine and cosine

$$\frac{\cos x}{\frac{1}{\cos x} \cdot \left(\frac{\cos x}{\sin x}\right)^2}$$

Simplify

$$\sin^2 x$$

■

$$7) \frac{\cot^2 x}{\sin x} = \frac{\csc x}{\tan^2 x}$$

$$\frac{\cot^2 x}{\sin x}$$

Use $\cot x = \frac{1}{\tan x}$

$$\frac{1}{\tan^2 x \sin x}$$

Use $\csc x = \frac{1}{\sin x}$

$$\frac{\csc x}{\tan^2 x}$$

■

$$8) \tan x \csc x = \frac{1}{\cos x}$$

$$\tan x \csc x$$

Decompose into sine and cosine

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$$

Simplify

$$\frac{1}{\cos x}$$

■

$$9) \sec x \cdot (1 + \sec x) = \frac{1 + \cos x}{\cos^2 x}$$

$$\sec x \cdot (1 + \sec x) \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\cos x} \left(1 + \frac{1}{\cos x} \right) \quad \text{Simplify}$$

$$\frac{1 + \cos x}{\cos^2 x} \quad \blacksquare$$

$$10) \frac{\sec^2 x}{\tan^2 x - 1} = \frac{1}{\sin^2 x - \cos^2 x}$$

$$\frac{\sec^2 x}{\tan^2 x - 1} \quad \text{Decompose into sine and cosine}$$

$$\frac{\left(\frac{1}{\cos x} \right)^2}{\left(\frac{\sin x}{\cos x} \right)^2 - 1} \quad \text{Simplify}$$

$$\frac{1}{\sin^2 x - \cos^2 x} \quad \blacksquare$$

$$11) \frac{\sin x + \csc x}{\csc x} = 1 + \sin^2 x$$

$$\frac{\sin x + \csc x}{\csc x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\sin x + \frac{1}{\sin x}}{\frac{1}{\sin x}} \quad \text{Simplify}$$

$$1 + \sin^2 x \quad \blacksquare$$

$$12) \frac{\csc x + 1}{\csc^2 x} = \sin x \cdot (\sin x + 1)$$

$$\frac{\csc x + 1}{\csc^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\sin x} + 1}{\left(\frac{1}{\sin x} \right)^2} \quad \text{Simplify}$$

$$\sin x \cdot (\sin x + 1) \quad \blacksquare$$

$$13) \frac{1 - \csc x}{\csc x} = \sin x - 1$$

$$\frac{1 - \csc x}{\csc x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1 - \frac{1}{\sin x}}{\frac{1}{\sin x}} \quad \text{Simplify}$$

$$\sin x - 1 \quad \blacksquare$$

$$14) \tan^2 x \cot^3 x = \frac{\cos x}{\sin x}$$

$$\tan^2 x \cot^3 x \quad \text{Decompose into sine and cosine}$$

$$\left(\frac{\sin x}{\cos x}\right)^2 \cdot \left(\frac{\cos x}{\sin x}\right)^3 \quad \text{Simplify}$$

$$\frac{\cos x}{\sin x} \quad \blacksquare$$

$$15) \frac{\sec^2 x + \cos x}{\sec^2 x} = 1 + \cos^3 x$$

$$\frac{\sec^2 x + \cos x}{\sec^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\left(\frac{1}{\cos x}\right)^2 + \cos x}{\left(\frac{1}{\cos x}\right)^2} \quad \text{Simplify}$$

$$1 + \cos^3 x \quad \blacksquare$$

$$16) 1 + \sec x = \frac{1 + \cos x}{\cos x}$$

$$1 + \sec x \quad \text{Decompose into sine and cosine}$$

$$1 + \frac{1}{\cos x} \quad \text{Simplify}$$

$$\frac{1 + \cos x}{\cos x} \quad \blacksquare$$

$$17) \frac{\cot x - 1}{\csc x} = \cos x - \sin x$$

$$\frac{\cot x - 1}{\csc x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{\cos x}{\sin x} - 1}{\frac{1}{\sin x}} \quad \text{Simplify}$$

$$\cos x - \sin x \quad \blacksquare$$

$$18) \frac{\csc x}{1 - \csc x} = \frac{1}{\sin x - 1}$$

$$\frac{\csc x}{1 - \csc x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\sin x}}{1 - \frac{1}{\sin x}} \quad \text{Simplify}$$

$$\frac{1}{\sin x - 1} \quad \blacksquare$$

$$19) \frac{\sec x + 1}{\sec x} = 1 + \cos x$$

$$\frac{\sec x + 1}{\sec x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\cos x} + 1}{\frac{1}{\cos x}} \quad \text{Simplify}$$

$$1 + \cos x \quad \blacksquare$$

$$20) \frac{\sec x - \tan x}{\sec x} = 1 - \sin x$$

$$\frac{\sec x - \tan x}{\sec x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \quad \text{Simplify}$$

$$1 - \sin x \quad \blacksquare$$

$$21) \frac{\tan^2 x - \sec^2 x}{\csc x} = -\sin x$$

$$\frac{\tan^2 x - \sec^2 x}{\csc x} \quad \text{Use } \tan^2 x + 1 = \sec^2 x$$

$$-\frac{1}{\csc x} \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$-\sin x \quad \blacksquare$$

$$22) \sec^2 x - \csc^2 x = \tan^2 x - \cot^2 x$$

$$\sec^2 x - \csc^2 x \quad \text{Use } \cot^2 x + 1 = \csc^2 x$$

$$\sec^2 x - \cot^2 x - 1 \quad \text{Use } \tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x - \cot^2 x \quad \blacksquare$$

$$23) \frac{\sec x + 1}{\sec^2 x} = \cos x \cdot (1 + \cos x)$$

$$\frac{\sec x + 1}{\sec^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\cos x} + 1}{\left(\frac{1}{\cos x}\right)^2} \quad \text{Simplify}$$

$$\cos x \cdot (1 + \cos x) \quad \blacksquare$$

$$24) \frac{1 + \cot^2 x}{\sin x} = \csc^3 x$$

$$\frac{1 + \cot^2 x}{\sin x} \quad \text{Use } \cot^2 x + 1 = \csc^2 x$$

$$\frac{\csc^2 x}{\sin x} \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\csc^3 x \quad \blacksquare$$