

## Unit 4.1 Fundamental Identities Basic EXAMPLES

Verify each identity.

$$1) \sin x + \csc x = \frac{1 + \sin^2 x}{\sin x}$$

$$2) \csc x \cdot (\csc x + 1) = \frac{\sin x + 1}{\sin^2 x}$$

$$3) \frac{\cos x}{\csc^2 x} = \frac{\sin^2 x}{\sec x}$$

$$4) \frac{\cot^2 x}{\sin^2 x} = \frac{\csc^2 x}{\tan^2 x}$$

$$5) -\csc x \cos x = -\cot x$$

$$6) \csc x \cdot (1 - \csc x) = \frac{\sin x - 1}{\sin^2 x}$$

$$7) \frac{\cot^2 x - \cos x}{\cot^2 x} = \frac{\cos x - \sin^2 x}{\cos x}$$

$$8) \sec^2 x + \tan^2 x = \frac{\sin^2 x + 1}{\cos^2 x}$$

$$9) \frac{1 + \csc x}{\csc^2 x} = \sin x \cdot (\sin x + 1)$$

$$10) \frac{1}{\sec^2 x - 1} = \frac{\cos x}{\sin x \tan x}$$

$$11) \sin x \sec x = \frac{1}{\cot x}$$

$$12) -\sin x \cot x = -\cos x$$

$$13) \frac{\csc x}{\cos^2 x} = \frac{\sec^2 x}{\sin x}$$

$$14) \frac{\sin^2 x}{\cos^2 x \tan x} = \tan^2 x \cot x$$

$$15) \frac{\sec x + \sin^2 x}{\sec x} = 1 + \sin^2 x \cos x$$

$$16) \frac{1}{\sec^2 x \cot x} = \cos x \sin x$$

$$17) \sin x - \cot x = \frac{\sin^2 x - \cos x}{\sin x}$$

$$18) \frac{1}{\tan^2 x \sec x} = \cot^2 x \cos x$$

$$19) \tan^2 x + \sec x = \frac{\sin^2 x + \cos x}{\cos^2 x}$$



$$20) \frac{\cos^2 x}{\tan^2 x} = \frac{\cot^2 x}{\sec^2 x}$$

$$21) \frac{1}{\sec x \cot^2 x} = \frac{\sin^2 x}{\cos x}$$

$$22) \sec^2 x \sin x = \frac{\tan x}{\cos x}$$

$$23) \frac{1}{\csc x + \sin x} = \frac{\sin x}{1 + \sin^2 x}$$

$$24) \csc x - 1 = \frac{1 - \sin x}{\sin x}$$

## Unit 4.1 Fundamental Identities Basic EXAMPLES

Verify each identity.

$$1) \sin x + \csc x = \frac{1 + \sin^2 x}{\sin x}$$

 $\sin x + \csc x$     Decompose into sine and cosine

$$\sin x + \frac{1}{\sin x}$$
    Simplify

$$\frac{1 + \sin^2 x}{\sin x} \quad \blacksquare$$

$$2) \csc x \cdot (\csc x + 1) = \frac{\sin x + 1}{\sin^2 x}$$

 $\csc x \cdot (\csc x + 1)$     Decompose into sine and cosine

$$\frac{1}{\sin x} \left( \frac{1}{\sin x} + 1 \right)$$
    Simplify

$$\frac{\sin x + 1}{\sin^2 x} \quad \blacksquare$$

$$3) \frac{\cos x}{\csc^2 x} = \frac{\sin^2 x}{\sec x}$$

$$\frac{\cos x}{\csc^2 x} \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\sin^2 x \cos x \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{\sin^2 x}{\sec x} \quad \blacksquare$$

$$4) \frac{\cot^2 x}{\sin^2 x} = \frac{\csc^2 x}{\tan^2 x}$$

$$\frac{\cot^2 x}{\sin^2 x} \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\cot^2 x \csc^2 x \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\frac{\csc^2 x}{\tan^2 x} \quad \blacksquare$$

$$5) -\csc x \cos x = -\cot x$$

$$-\csc x \cos x \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$-\frac{\cos x}{\sin x} \quad \text{Use } \cot x = \frac{\cos x}{\sin x}$$

$$-\cot x \quad \blacksquare$$

$$6) \csc x \cdot (1 - \csc x) = \frac{\sin x - 1}{\sin^2 x}$$

$$\csc x \cdot (1 - \csc x) \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\sin x} \left( 1 - \frac{1}{\sin x} \right) \quad \text{Simplify}$$

$$\frac{\sin x - 1}{\sin^2 x} \quad \blacksquare$$

$$7) \frac{\cot^2 x - \cos x}{\cot^2 x} = \frac{\cos x - \sin^2 x}{\cos x}$$

$$\frac{\cot^2 x - \cos x}{\cot^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\left( \frac{\cos x}{\sin x} \right)^2 - \cos x}{\left( \frac{\cos x}{\sin x} \right)^2} \quad \text{Simplify}$$

$$\frac{\cos x - \sin^2 x}{\cos x} \quad \blacksquare$$

$$8) \sec^2 x + \tan^2 x = \frac{\sin^2 x + 1}{\cos^2 x}$$

$$\sec^2 x + \tan^2 x \quad \text{Decompose into sine and cosine}$$

$$\left(\frac{1}{\cos x}\right)^2 + \left(\frac{\sin x}{\cos x}\right)^2 \quad \text{Simplify}$$

$$\frac{\sin^2 x + 1}{\cos^2 x} \quad \blacksquare$$

$$9) \frac{1 + \csc x}{\csc^2 x} = \sin x \cdot (\sin x + 1)$$

$$\frac{1 + \csc x}{\csc^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1 + \frac{1}{\sin x}}{\left(\frac{1}{\sin x}\right)^2} \quad \text{Simplify}$$

$$\sin x \cdot (\sin x + 1) \quad \blacksquare$$

$$10) \frac{1}{\sec^2 x - 1} = \frac{\cos x}{\sin x \tan x}$$

$$\frac{1}{\sec^2 x - 1} \quad \text{Use } \tan^2 x + 1 = \sec^2 x$$

$$\frac{1}{\tan^2 x} \quad \text{Use } \tan x = \frac{\sin x}{\cos x}$$

$$\frac{\cos x}{\sin x \tan x} \quad \blacksquare$$

$$11) \sin x \sec x = \frac{1}{\cot x}$$

$$\sin x \sec x \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos x} \quad \text{Use } \cot x = \frac{\cos x}{\sin x}$$

$$\frac{1}{\cot x} \quad \blacksquare$$

$$12) -\sin x \cot x = -\cos x$$

$$-\sin x \cot x \quad \text{Use } \cot x = \frac{\cos x}{\sin x}$$

$$-\frac{\sin x \cos x}{\sin x} \quad \text{Cancel common factors}$$

$$-\cos x \quad \blacksquare$$

$$13) \frac{\csc x}{\cos^2 x} = \frac{\sec^2 x}{\sin x}$$

$$\frac{\csc x}{\cos^2 x} \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\frac{1}{\cos^2 x \sin x} \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{\sec^2 x}{\sin x} \quad \blacksquare$$

$$14) \frac{\sin^2 x}{\cos^2 x \tan x} = \tan^2 x \cot x$$

$$\frac{\sin^2 x}{\cos^2 x \tan x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\frac{\sin^2 x \cot x}{\cos^2 x} \quad \text{Use } \tan x = \frac{\sin x}{\cos x}$$

$$\tan^2 x \cot x \quad \blacksquare$$



$$15) \frac{\sec x + \sin^2 x}{\sec x} = 1 + \sin^2 x \cos x$$

$$\frac{\sec x + \sin^2 x}{\sec x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\frac{1}{\cos x} + \sin^2 x}{\frac{1}{\cos x}} \quad \text{Simplify}$$

$$1 + \sin^2 x \cos x \quad \blacksquare$$

$$16) \frac{1}{\sec^2 x \cot x} = \cos x \sin x$$

$$\frac{1}{\sec^2 x \cot x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\left(\frac{1}{\cos x}\right)^2 \cdot \frac{\cos x}{\sin x}} \quad \text{Simplify}$$

$$\cos x \sin x \quad \blacksquare$$

$$17) \sin x - \cot x = \frac{\sin^2 x - \cos x}{\sin x}$$

$$\sin x - \cot x \quad \text{Decompose into sine and cosine}$$

$$\sin x - \frac{\cos x}{\sin x} \quad \text{Simplify}$$

$$\frac{\sin^2 x - \cos x}{\sin x} \quad \blacksquare$$

$$18) \frac{1}{\tan^2 x \sec x} = \cot^2 x \cos x$$

$$\frac{1}{\tan^2 x \sec x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\frac{\cot^2 x}{\sec x} \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\cot^2 x \cos x \quad \blacksquare$$

$$19) \tan^2 x + \sec x = \frac{\sin^2 x + \cos x}{\cos^2 x}$$

$$\tan^2 x + \sec x \quad \text{Decompose into sine and cosine}$$

$$\left(\frac{\sin x}{\cos x}\right)^2 + \frac{1}{\cos x} \quad \text{Simplify}$$

$$\frac{\sin^2 x + \cos x}{\cos^2 x} \quad \blacksquare$$

$$20) \frac{\cos^2 x}{\tan^2 x} = \frac{\cot^2 x}{\sec^2 x}$$

$$\frac{\cos^2 x}{\tan^2 x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\cos^2 x \cot^2 x \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{\cot^2 x}{\sec^2 x} \quad \blacksquare$$

$$21) \frac{1}{\sec x \cot^2 x} = \frac{\sin^2 x}{\cos x}$$

$$\frac{1}{\sec x \cot^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\frac{1}{\cos x} \cdot \left(\frac{\cos x}{\sin x}\right)^2} \quad \text{Simplify}$$

$$\frac{\sin^2 x}{\cos x} \quad \blacksquare$$

$$22) \sec^2 x \sin x = \frac{\tan x}{\cos x}$$

$$\sec^2 x \sin x \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos^2 x} \quad \text{Use } \tan x = \frac{\sin x}{\cos x}$$

$$\frac{\tan x}{\cos x} \quad \blacksquare$$

$$23) \frac{1}{\csc x + \sin x} = \frac{\sin x}{1 + \sin^2 x}$$

$$\frac{1}{\csc x + \sin x} \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\frac{1}{\sin x} + \sin x} \quad \text{Simplify}$$

$$\frac{\sin x}{1 + \sin^2 x} \quad \blacksquare$$

$$24) \csc x - 1 = \frac{1 - \sin x}{\sin x}$$

$$\csc x - 1 \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\sin x} - 1 \quad \text{Simplify}$$

$$\frac{1 - \sin x}{\sin x} \quad \blacksquare$$