## Study Guide - Rules for Transformations on a Coordinate Plane

Translations: one type of transformation where a geometric figure is "slide" horizontally, vertically, or both. Sliding a polygon to a new position without turning it. When translating a figure, every point of the original figure is moved the same distance and in the same direction.

Rules: A positive integer describes a translation right or up on a coordinate plane.
A negative integer describes a translation left or down on a coordinate plane.
*A movement left or right is on the $\mathbf{x}$-axis. A movement up or down is on the $\mathbf{y}$-axis.
Example 1: Translate trapezoid HIJK 3 units left and 5 units up. This can also be written as $(-3,5)$, or $(x-3, y+5)$
Example 2: Translate triangle ABC 5 units left and 1 unit up. This can be written as $(-5,1)$, or $(x-5, y+1)$
Example 3: Trapezoid GHIJ has vertices $\mathrm{G}(-4,1), \mathrm{H}(-4,3), \mathrm{I}(-2,3)$, and $\mathrm{J}(-1,1)$. Find the vertices of trapezoid G'H'I'J' after a translation of 5 units right and 3 units down. Then graph the figure and its translated image.

$$
\begin{array}{llll}
\mathrm{G}(-4,1) & (x+5, y-3) & \mathrm{G}^{\prime}(-4+5,1-3) & \mathrm{G}^{\prime}(1,-2) \\
\mathrm{H}(-4,3) & (\mathrm{x}+5, \mathrm{y}-3) & \mathrm{H}^{\prime}(-4+5,3-3) & \mathrm{H}^{\prime}(1,0) \\
\mathrm{I}(-2,3) & (x+5, y-3) & \mathrm{I}^{\prime}(-2+5,3-3) & \mathrm{I}^{\prime}(3,0) \\
\mathrm{J}(-1,1) & (x+5, y-3) & \mathrm{J}^{\prime}(-1+5,1-3) & \mathrm{J}^{\prime}(4,-2)
\end{array}
$$

Reflections: A type of transformation where a figure is "flipped" over a line of symmetry. A reflection produces a mirror image of a figure.

Rules: Reflect a figure over the $\mathbf{x}$-axis- when reflecting over the x -axis, change the y coordinates to their opposites. (x, -y)

Reflect a figure over the $y$-axis- when reflecting over the $y$-axis, change the $x$ coordinates to their opposites. (-x, y)

Example 1: Triangle ABC has vertices $\mathrm{A}(5,2), \mathrm{B}(1,3)$, and $\mathrm{C}(-1,1)$. Find the coordinates of ABC after a reflection over the x -axis.

| $\mathrm{A}(5,2)$ | $(x,-y)$ | $A^{\prime}(5,-2)$ |
| :--- | :--- | :--- |
| $\mathrm{B}(1,3)$ | $(x,-y)$ | $\mathrm{B}^{\prime}(1,-3)$ |
| $\mathrm{C}(-1,1)$ | $(x,-y)$ | $C^{\prime}(-1,-1)$ |

Example 2: Quadrilateral KLMN has vertices $\mathrm{K}(2,3)$, $\mathrm{L}(5,1)$, $\mathrm{M}(4,-2)$, and $\mathrm{N}(1,-1)$. Find the coordinates of KLMN after a reflection over the $y$-axis. Then graph the figure and its reflected image.

| $\mathrm{K}(2,3)$ | $(-x, y)$ | $\mathrm{K}^{\prime}(-2,3)$ |
| :--- | :--- | :--- |
| $\mathrm{L}(5,1)$ | $(-x, y)$ | $\mathrm{L}^{\prime}(-5,1)$ |
| $\mathrm{M}(4,-2)$ | $(-x, y)$ | $\mathrm{M}^{\prime}(-4,-2)$ |
| $\mathrm{N}(1,-1)$ | $(-x, y)$ | $\mathrm{N}^{\prime}(-1,-1)$ |

Rotations: A transformation that "turns" a figure about a fixed point at a given angle and a given direction.
Rules: 90 degree clockwise rotation around the origin ( 0,0 ), use: ( $\mathbf{y},-\mathbf{x}$ )
180 degree rotation around the origin ( 0,0 ), use: (-x, -y)
270 degree clockwise rotation around the origin ( 0,0 ), use: $(-\mathbf{y}, \mathbf{x})$
Example 1: Triangle NPQ has vertices $\mathrm{N}(0,0), \mathrm{P}(4,-1)$, and $\mathrm{Q}(4,2)$. Rotate clockwise 90 degrees.

| $\mathrm{N}(0,0)$ | $(y,-x)$ | $\mathrm{N}^{\prime}(0,0)$ |
| :--- | :--- | :--- |
| $\mathrm{P}(4,-1)$ | $(y,-x)$ | $\mathrm{P}^{\prime}(-1,-4)$ |
| $\mathrm{Q}(4,2)$ | $(y,-x)$ | $\mathrm{Q}^{\prime}(2,-4)$ |

Example 2: Triangle KLM has vertices $\mathrm{K}(1,0), \mathrm{L}(4,2)$, and $\mathrm{M}(3,4)$. Rotate 180 degrees.

| $\mathrm{K}(1,0)$ | $(-x,-y)$ | $\mathrm{K}^{\prime}(-1,0)$ |
| :--- | :--- | :--- |
| $\mathrm{L}(4,2)$ | $(-x,-y)$ | $L^{\prime}(-4,-2)$ |
| $\mathrm{M}(3,4)$ | $(-x,-y)$ | $\mathrm{M}^{\prime}(-3,-4)$ |

Example 3: Quadrilateral DEFG has vertices D(-1,0), $\mathrm{E}(-4,1), \mathrm{F}(-3,3)$, and $\mathrm{G}(0,4)$. Rotate clockwise 270 degrees. Graph DEFG and D'E'F'G'.

| $D(-1,0)$ | $(-y, x)$ | $D^{\prime}(0,-1)$ |
| :--- | :--- | :--- |
| $E(-4,1)$ | $(-y, x)$ | $E^{\prime}(-1,-4)$ |
| $F(-3,3)$ | $(-y, x)$ | $F^{\prime}(-3,-3)$ |
| $G(0,4)$ | $(-y, x)$ | $G^{\prime}(-4,0)$ |

Dilations: a transformation that changes the size of a figure, but not the shape.
Rule: To dilate a figure, always MULTIPLY the coordinates of each of its points by the percent of dilation.
**First change the percent to a decimal (move the decimal point TWO places to the LEFT.
**Next, multiply each of the coordinates by that number.
Example 1: Triangle ABC has vertices $\mathrm{A}(-2,2), \mathrm{B}(-1,-2), \mathrm{C}(-6,1)$. What are the new coordinates after a dilation of $150 \%$ ?

Change the percent to a decimal: $150 \%=1.50$
$\mathrm{A}^{\prime}(-2 \times 1.5,2 \times 1.5) \quad \mathrm{B}^{\prime}(-1 \times 1.5,-2 \times 1.5) \mathrm{C}^{\prime}(-6 \times 1.5,1 \times 1.5)$
$\mathrm{A}^{\prime}(-3,3) \mathrm{B}^{\prime}(-1.5,-3) \mathrm{C}^{\prime}(-9,1.5)$
Example 2: Triangle XYZ has vertices $\mathrm{X}(-4,3), \mathrm{Y}(2,3), \mathrm{Z}(-3,1)$. What are the new coordinates after a dilation of $75 \%$ ?

Change the percent to a decimal, then multiply: $\mathbf{7 5 \%}=\mathbf{. 7 5}$

$$
X^{\prime}(-4 \times .75,3 \times .75) \quad Y^{\prime}(2 \times .75,3 \times .75) \quad Z^{\prime}(-3 \times .75,1 \times .75)
$$

$X^{\prime}(-3,2.25) Y^{\prime}(1.5,2.25) Z^{\prime}(-2.25, .75)$
Example 3: Triangle XYZ has vertices $X(12,20), \mathrm{Y}(24,4), \mathrm{Z}(4,16)$. If the new coordinates after a dilation are $X^{\prime}(3,5), Y^{\prime}(6,1), Z^{\prime}(1,4)$, what was the percent of dilation?

Rule: Divide the coordinates of the image by the coordinates of the original figure to determine the percent of dilation.

X (3/12, 5/20) Y (6/24, 1/4) Z(1/4, 4/16)
Percent of Dilation: 25\%

