Notes 10-1A Simplifying Radical Expressions

$\sqrt{\ }$

I. Product Property of Square Roots

KEY CONCEPT ***Product Property of Square Roots***

Words For any numbers a and b, where $a \geq 0$ and $b \geq 0$, the square root of the product ab is equal to the product of each square root.

Symbols $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ **Example** $\sqrt{4 \cdot 25} = \sqrt{4} \cdot \sqrt{25}$

A. Simplifying simple radical expressions

Method 1: Perfect Square Method
-Break the radicand into perfect square(s) and simplify.

Ex 1: $\sqrt{72}$

$$= \sqrt{36 \cdot 2}$$
$$= \sqrt{36} \cdot \sqrt{2}$$
$$\boxed{= 6\sqrt{2}}$$

Ex 2: $\sqrt{48}$

$$= \sqrt{16 \cdot 3}$$
$$= \sqrt{16} \cdot \sqrt{3}$$
$$\boxed{= 4\sqrt{3}}$$

Simplify

Perfect Square Factor * Other Factor

LEAVE IN RADICAL FORM

Ex 3: $\sqrt{80} = \sqrt{16*5} = 4\sqrt{5}$

Ex 4: $\sqrt{50} = \sqrt{25*2} = 5\sqrt{2}$

Ex 5: $\sqrt{125} = \sqrt{25*5} = 5\sqrt{5}$

Ex 6: $\sqrt{450} = \sqrt{225*2} = 15\sqrt{2}$

Method 2: Pair Method

Sometimes it is difficult to recognize perfect squares within a number. You will get better at it with more practice, but until then, here is a second method:

-Break the radicand up into prime factors
-group pairs of the same number
-simplify
-multiply any numbers in front of the radical; multiply any numbers inside of the radical

Example 1:

Ex 1: Simplify $\sqrt{72}$

$$\sqrt{2*2*2*3*3}$$

Step 1: Break up into prime factors

$$\sqrt{2*2}*\sqrt{3*3}*\sqrt{2}$$

Step 2: Group together any pairs

$$\sqrt{4}\sqrt{9}\sqrt{2}$$

Step 3: Simplify

$$2*3\sqrt{2}$$

$$\boxed{=6\sqrt{2}}$$

Step 4: Multiply numbers in front of radical; multiply numbers inside radical

Example 2:

Ex 2: Simplify $\sqrt{48}$

$$\sqrt{2*2*2*2*3}$$

Step 1: Break up into prime factors

$$\sqrt{2*2}*\sqrt{2*2}*\sqrt{3}$$

Step 2: Group together any pairs

$$\sqrt{2^2}\sqrt{2^2}\sqrt{3}$$

Step 3: Simplify

$$2*2\sqrt{3}$$

$$=\boxed{4\cdot\sqrt{3}}$$

Step 4: Multiply numbers in front of radical; multiply numbers inside radical

More Examples:

Ex 3:

$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

Ex 4:

$$7\sqrt{75} = 7\sqrt{25 \cdot 3} = 7\sqrt{25}\sqrt{3} = 7 \cdot 5\sqrt{3} = 35\sqrt{3}$$

Simplify

LEAVE IN RADICAL FORM

Ex 5: $\sqrt{8} = \sqrt{4*2} = 2\sqrt{2}$

Ex 6: $\sqrt{20} = \sqrt{4*5} = 2\sqrt{5}$

Ex 7: $\sqrt{32} = \sqrt{16*2} = 4\sqrt{2}$

Ex 8: $\sqrt{75} = \sqrt{25*3} = 5\sqrt{3}$

Ex 9: $\sqrt{40} = \sqrt{4*10} = 2\sqrt{10}$

B. Using Product Property to Multiply Square Roots

Ex 1: Multiply $\sqrt{3} * \sqrt{15}$

Method 1: Break down and simplify

$$\sqrt{3} * \sqrt{15}$$

$$\sqrt{3} * \sqrt{3} * \sqrt{5}$$

$$\sqrt{3^2} * \sqrt{5}$$

$$3\sqrt{5}$$

Method 2: Multiply together first

$$\sqrt{3 * 15}$$

$$\sqrt{45}$$

$$\sqrt{9 * 5}$$

$$3\sqrt{5}$$

Practice

Example 2: Mulitply $\sqrt{5} * \sqrt{10}$

$$\sqrt{5} * \sqrt{10}$$

$$\sqrt{50}$$

$$\sqrt{2 * 25}$$

$$5\sqrt{2}$$

Example 3: Multiply $2\sqrt{6} * 3\sqrt{8}$

$$2 * 3 * \sqrt{6} * \sqrt{8}$$

$$6\sqrt{48}$$

$$6\sqrt{16 * 3}$$

$$6 * 4\sqrt{3}$$

$$24\sqrt{3}$$

II. Simplifying Radical Expressions with Variables

When finding the principal square root of an expression containing variables, be sure that the result is not negative. Consider the expression $\sqrt{x^2}$. It may seem that $\sqrt{x^2} = x$. Let's look at $x = -2$.

$$\sqrt{x^2} \stackrel{?}{=} x$$

$$\sqrt{(-2)^2} \stackrel{?}{=} -2 \quad \text{Replace } x \text{ with } -2.$$

$$\sqrt{4} \stackrel{?}{=} -2 \quad (-2)^2 = 4$$

$$2 \neq -2 \quad \sqrt{4} = 2$$

For radical expressions where the exponent of the variable inside the radical is even and the resulting simplified exponent is odd, you must use absolute value to ensure nonnegative results.

$\sqrt{x^2} = |x| \quad \sqrt{x^3} = |x|\sqrt{x} \quad \sqrt{x^4} = x^2 \quad \sqrt{x^5} = x^2\sqrt{x} \quad \sqrt{x^6} = \left|x^3\right|$

More Examples:

1. $\sqrt{30a^{34}} = \sqrt{a^{34}} \cdot \sqrt{30}$

$$= \boxed{\left|a^{17}\right| \sqrt{30}}$$

2. $\sqrt{54x^4 y^5 z^7} = \sqrt{9x^4 y^4 z^6} \cdot \sqrt{6yz}$

$$= \boxed{3x^2 y^2 \left|z^3\right| \sqrt{6yz}}$$

A. Examples

Remember!!!!!

For radical expressions where the exponent of the variable inside the radical is even and the resulting simplified exponent is odd, you must use absolute value to ensure nonnegative results.

3. **Simplify $\sqrt{40x^4y^5z^3}$.**

$$\sqrt{40x^4y^5z^3} = \sqrt{2^3 \cdot 5 \cdot x^4 \cdot y^5 \cdot z^3}$$ Prime factorization

$$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{x^4} \cdot \sqrt{y^4} \cdot \sqrt{y} \cdot \sqrt{z^2} \cdot \sqrt{z}$$ Product Property

$$= 2 \cdot \sqrt{2} \cdot \sqrt{5} \cdot x^2 \cdot y^2 \cdot \sqrt{y} \cdot |z| \cdot \sqrt{z}$$ Simplify.

$$= 2x^2y^2|z|\sqrt{10yz}$$ The absolute value of *z* ensures a nonnegative result.

More Examples

Ex 4: $\sqrt{x^{11}} = \sqrt{x^{10} \cdot x} = \quad x^5\sqrt{x}$

Ex 5: $\sqrt{18x^4} = \sqrt{9 \cdot 2x^4} = \quad 3x^2\sqrt{2}$

III. Quotient Rule for Square Roots

If $\sqrt{a}$ and $\sqrt{b}$ are real numbers and $b \neq 0$, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples:

Ex 1: $\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9}$

Ex 2: $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

Ex 3: $\sqrt{\frac{45}{49}} = \frac{\sqrt{45}}{\sqrt{49}} = \frac{\sqrt{9 \cdot 5}}{7} = \frac{3\sqrt{5}}{7}$

Ex 4:

$$\frac{\sqrt{15}}{\sqrt{3}} = \frac{\sqrt{3\cdot 5}}{\sqrt{3}} = \frac{\sqrt{3}\cdot\sqrt{5}}{\sqrt{3}} = \sqrt{5}$$

Ex 5:

$$\frac{\sqrt{90}}{\sqrt{2}} = \frac{\sqrt{9\cdot 10}}{\sqrt{2}} = \frac{\sqrt{9\cdot 2\cdot 5}}{\sqrt{2}} = \frac{\sqrt{9}\cdot\sqrt{2}\cdot\sqrt{5}}{\sqrt{2}} = 3\sqrt{5}$$

Examples:

1. $$\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}} = \boxed{\frac{\sqrt{7}}{4}}$$

2. $$\sqrt{\frac{32}{25}} = \frac{\sqrt{32}}{\sqrt{25}} = \frac{\sqrt{32}}{5} = \boxed{\frac{4\cdot\sqrt{2}}{5}}$$

Ex 3: Simplify $\sqrt{\frac{48}{3}}$

Ex 4: Simplify

$$\frac{\sqrt{48}}{\sqrt{3}}$$

$$\frac{\sqrt{16 * 3}}{\sqrt{3}}$$

$$\frac{4\sqrt{3}}{\sqrt{3}}$$

$$\boxed{4}$$

Ex 4:

$$\sqrt{\frac{27}{x^8}} = \frac{\sqrt{27}}{\sqrt{x^8}} = \frac{\sqrt{9 \cdot 3}}{\sqrt{x^8}} = \frac{3\sqrt{3}}{x^4}$$

Ex 5:

$$\sqrt{\frac{7y^7}{25}} = \frac{\sqrt{7 \cdot y^6 y}}{\sqrt{25}} = \frac{y^3\sqrt{7y}}{5}$$

IV. Rationalizing the Denominator

Radical Expressions are fully simplified when:

- There are no prime factors with an exponent greater than one under any radicals
- There are no fractions under any radicals
- There are no radicals in the denominator

Rationalizing the Denominator is a way to get rid of any radicals in the denominator

A. Denominators with one term (which is a radical)

- To rationalize the denominator of a quotient with a denominator of one term, multiply numerator and denominator by that term.

Simplify.

Ex 1: **a.** $\sqrt{\frac{10}{3}}$

$$\sqrt{\frac{10}{3}} = \frac{\sqrt{10}}{\sqrt{3}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$$= \frac{\sqrt{30}}{3}$$ Product Property of Square Roots

Ex 1: **b.** $\frac{\sqrt{2n}}{\sqrt{6}}$

$$\frac{\sqrt{2n}}{\sqrt{6}} = \frac{\sqrt{2n}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$ Multiply by $\frac{\sqrt{6}}{\sqrt{6}}$.

$$= \frac{\sqrt{12n}}{6}$$ Product Property of Square Roots

$$= \frac{\sqrt{2 \cdot 2 \cdot 3 \cdot n}}{6}$$ Prime factorization

$$= \frac{2\sqrt{3n}}{6}$$ $\sqrt{2^2} = 2$

$$= \frac{\sqrt{3n}}{3}$$ Divide numerator and denominator by 2.