Trigonometric Functions

Graphing the Trigonometric Function

Topic:
Graphing Trigonometric Functions

Objective(s):
Students will be able to graph trigonometric functions by finding the amplitude and period of variation of the sine cosine and tangent functions.

Essential Question(s):
1. What is a radian and how do I use it to determine angle measure on a circle?
2. How do I use trigonometric functions to model periodic behavior?

CCSS: F.IF. 2, 4, 5 &7E; f.tf. 1,2,5 &8

Mathematical Practices:

- **1. Make sense of problems and persevere in solving them.**
- **2. Reason abstractly and quantitatively.**
- **3. Construct viable arguments and critique the reasoning of others.**
- **4. Model with mathematics.**
- **5. Use appropriate tools strategically.**
- **6. Attend to precision.**
- **7. Look for and make use of structure.**
- **8. Look for and express regularity in repeated reasoning.**

SOH
CAH
TOA

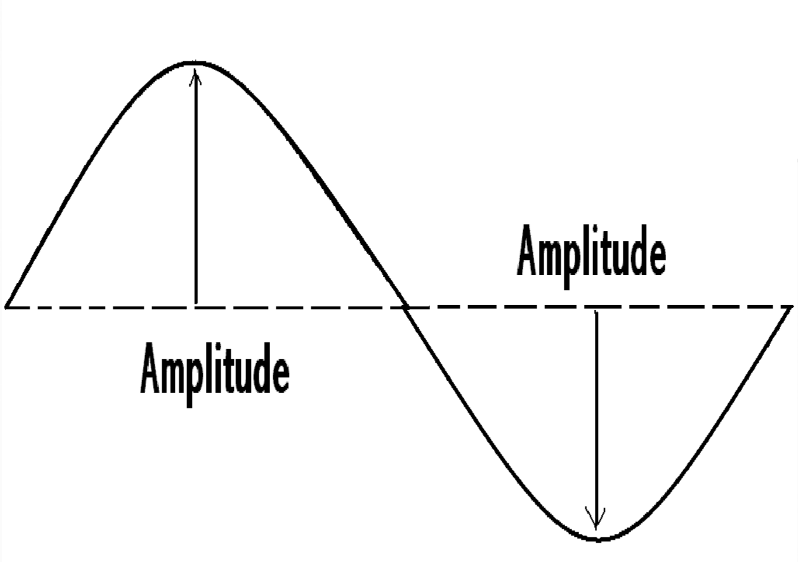
CHO
SHA
CAO

angle Trigonometry

Graphing the Trig Function

Graphing Trigonometric Functions

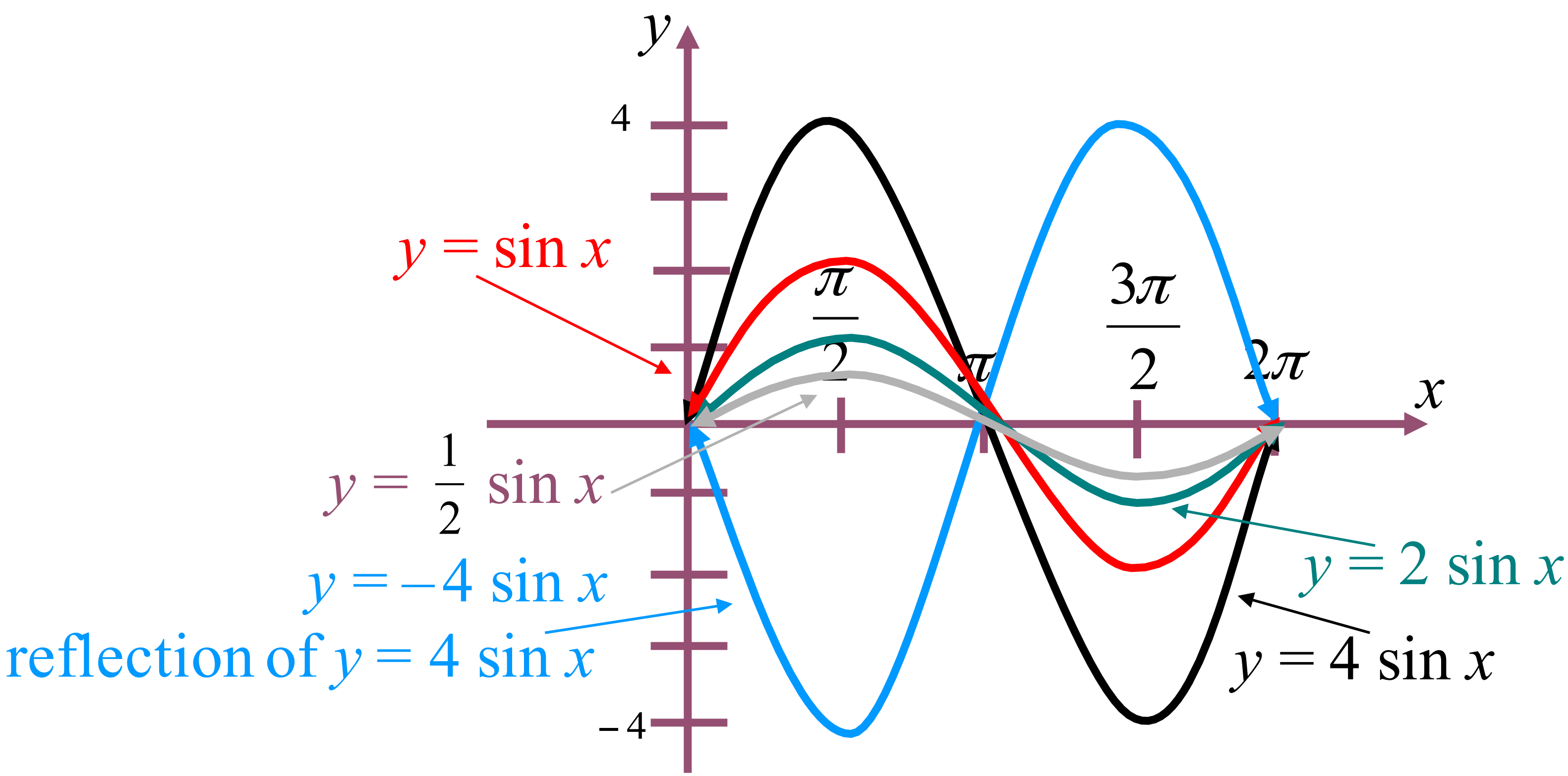
- Amplitude: the maximum or minimum vertical distance between the graph and the x-axis. Amplitude is always positive



The **amplitude** of $y = a \sin x$ (or $y = a \cos x$) is half the distance between the maximum and minimum values of the function.

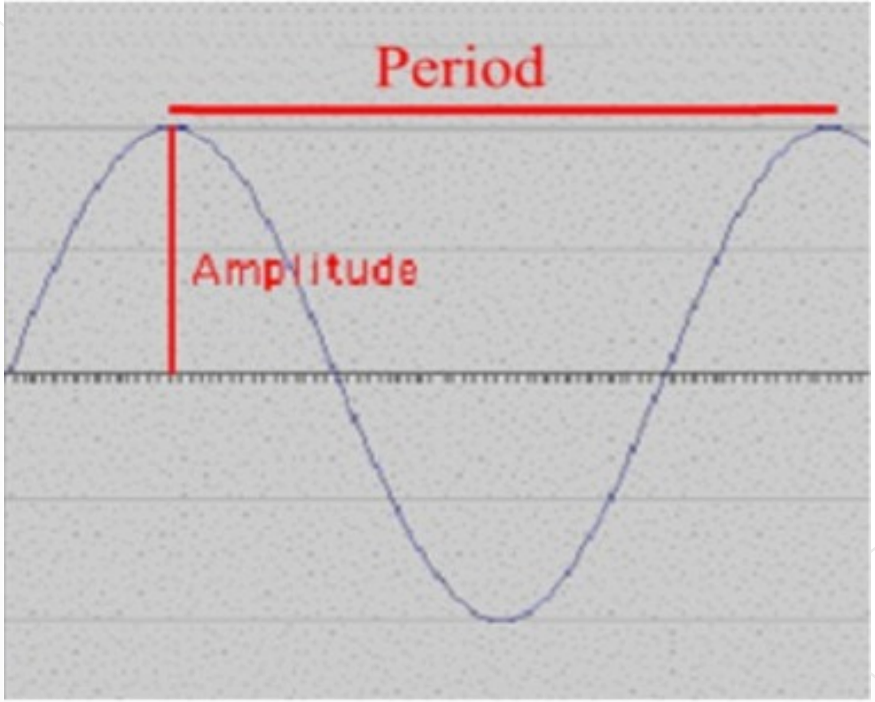
$$\text{amplitude} = |a|$$

If $|a| > 1$, the amplitude stretches the graph vertically.
If $0 < |a| > 1$, the amplitude shrinks the graph vertically.
If $a < 0$, the graph is reflected in the x-axis.



Graphing Trigonometric Functions

- Period: the number of degrees or radians we must graph before it begins again.

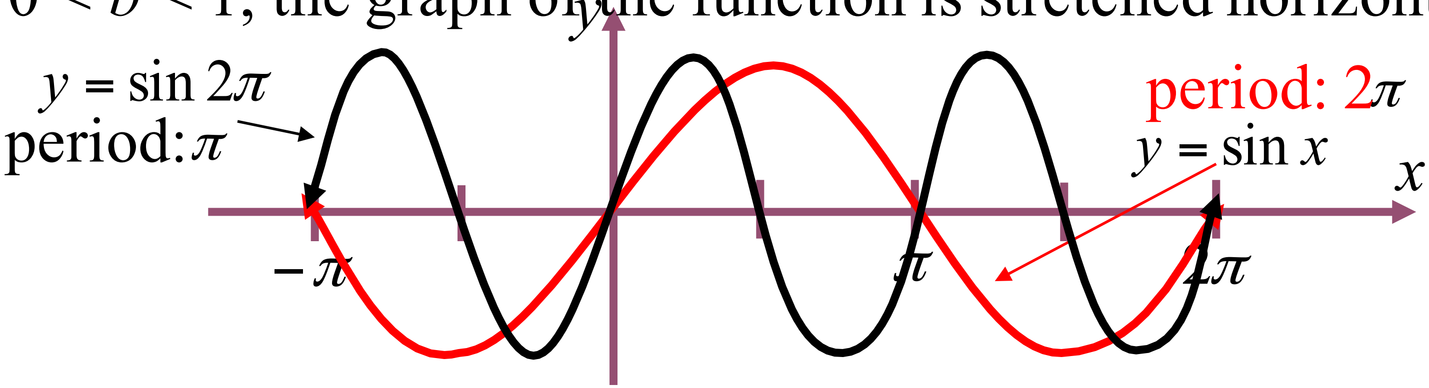


The **period** of a function is the x interval needed for the function to complete one cycle.

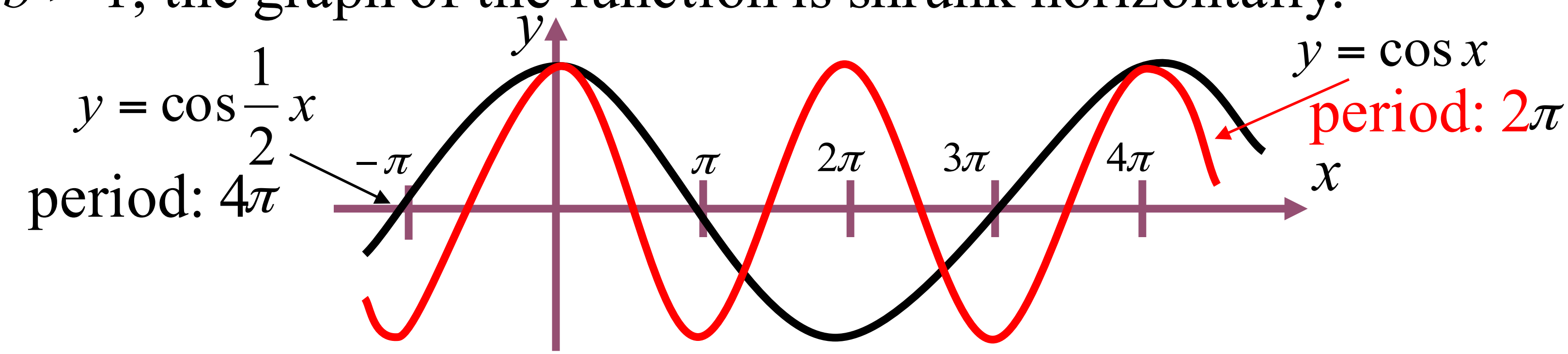
For $b > 0$, the **period** of $y = a \sin bx$ is $\frac{2\pi}{b}$.

For $b > 0$, the **period** of $y = a \cos bx$ is also $\frac{2\pi}{b}$.

If $0 < b < 1$, the graph of the function is stretched horizontally.



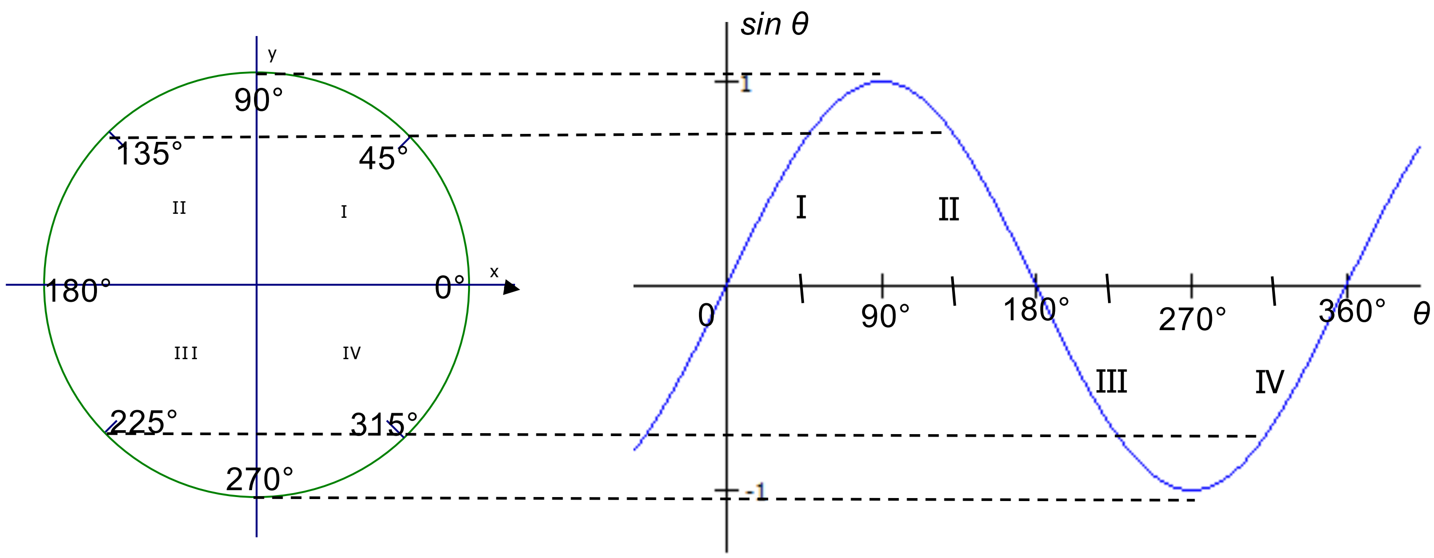
If $b > 1$, the graph of the function is shrunk horizontally.



The sine function

Imagine a particle on the unit circle, starting at (1,0) and rotating counterclockwise around the origin. Every position of the particle corresponds with an angle, θ, where y = sin θ. As the particle moves through the four quadrants, we get four pieces of the sin graph:

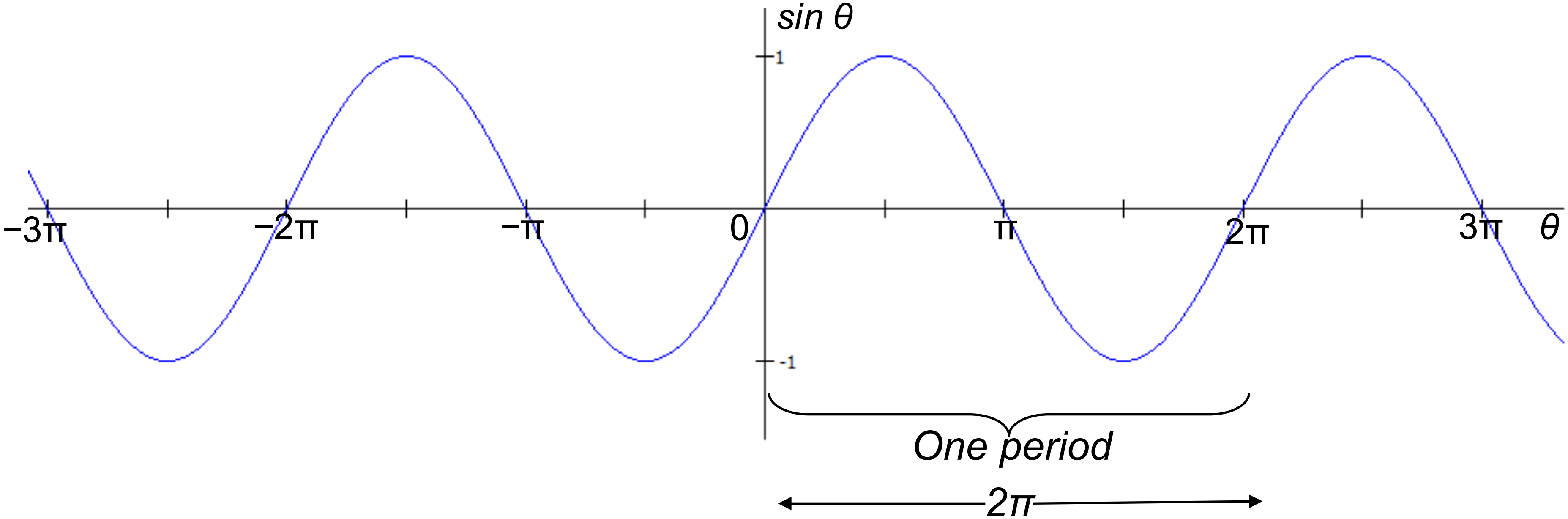
I. From 0° to 90° the y-coordinate increases from 0 to 1
II. From 90° to 180° the y-coordinate decreases from 1 to 0
III. From 180° to 270° the y-coordinate decreases from 0 to −1
IV. From 270° to 360° the y-coordinate increases from −1 to 0



θ	**sin θ**
0	0
π/2	1
π	0
3π/2	−1
2π	0

Interactive Sine Unwrap

Sine is a periodic function: p = 2π



sin θ: Domain (angle measures): all real numbers, (−∞, ∞)
Range (ratio of sides): −1 to 1, inclusive [−1, 1]

sin θ is an **odd** function; it is symmetric wrt the origin.

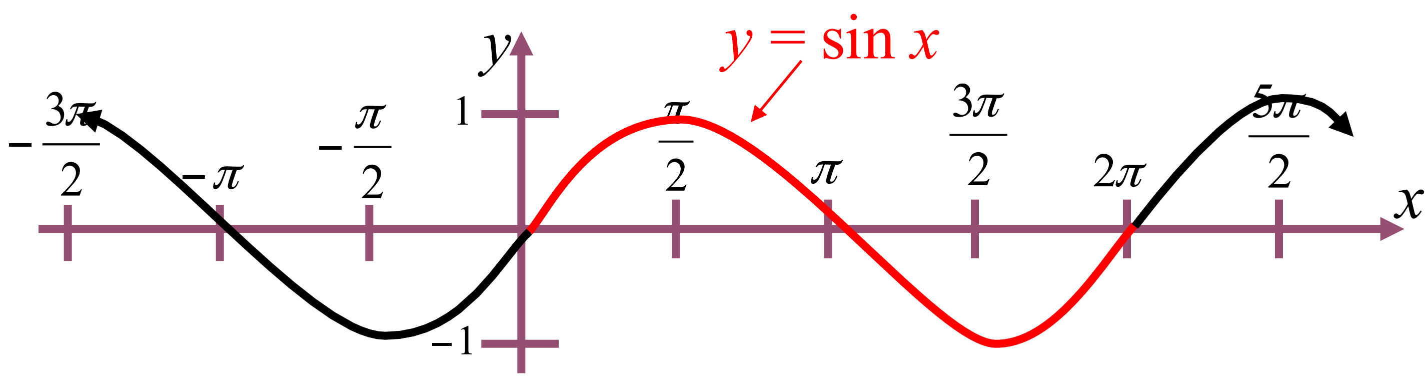
$$\forall \theta \in Domain, \ \sin(-\theta) = -\sin(\theta)$$

Graph of the Sine Function

To sketch the graph of $y = \sin x$ first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0

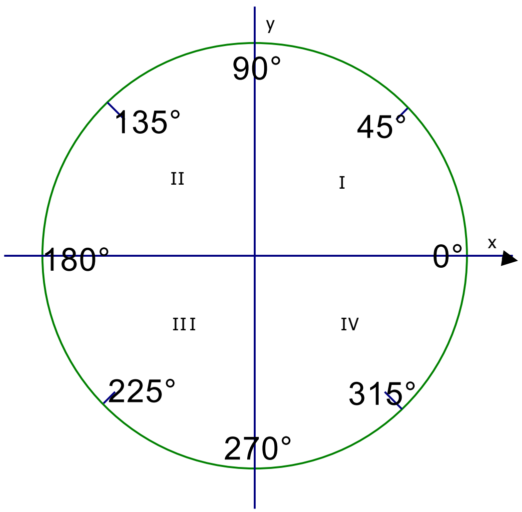
Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.

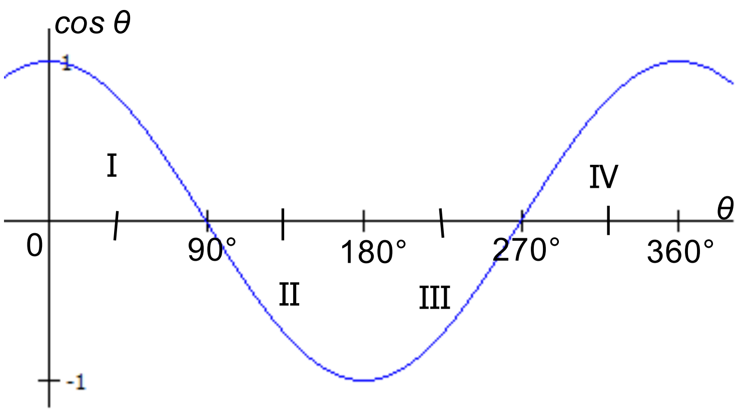


The cosine function

Imagine a particle on the unit circle, starting at (1,0) and rotating counterclockwise around the origin. Every position of the particle corresponds with an angle, θ, where x = cos θ. As the particle moves through the four quadrants, we get four pieces of the cos graph:

I. From 0° to 90° the x-coordinate decreases from 1 to 0
II. From 90° to 180° the x-coordinate decreases from 0 to −1
III. From 180° to 270° the x-coordinate increases from −1 to 0
IV. From 270° to 360° the x-coordinate increases from 0 to 1





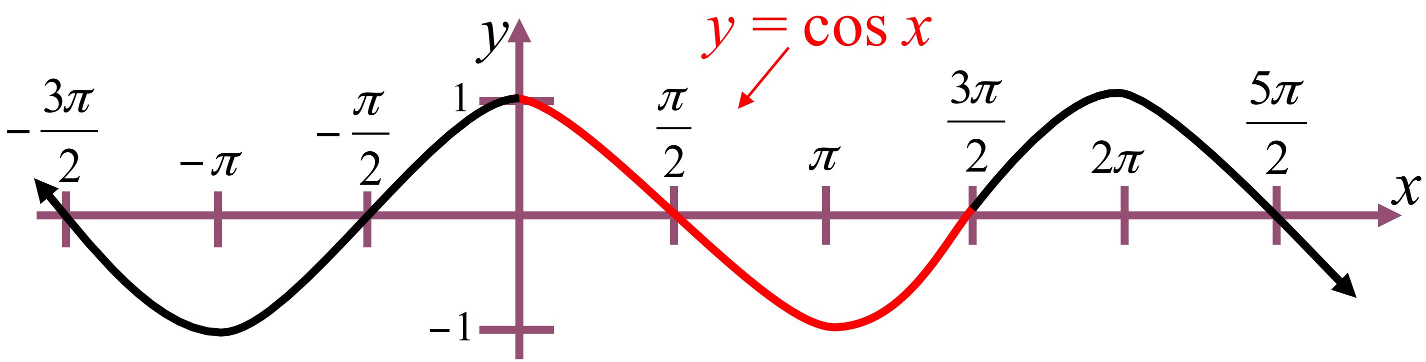
θ	cos θ
0	1
π/2	0
π	−1
3π/2	0
2π	1

Graph of the Cosine Function

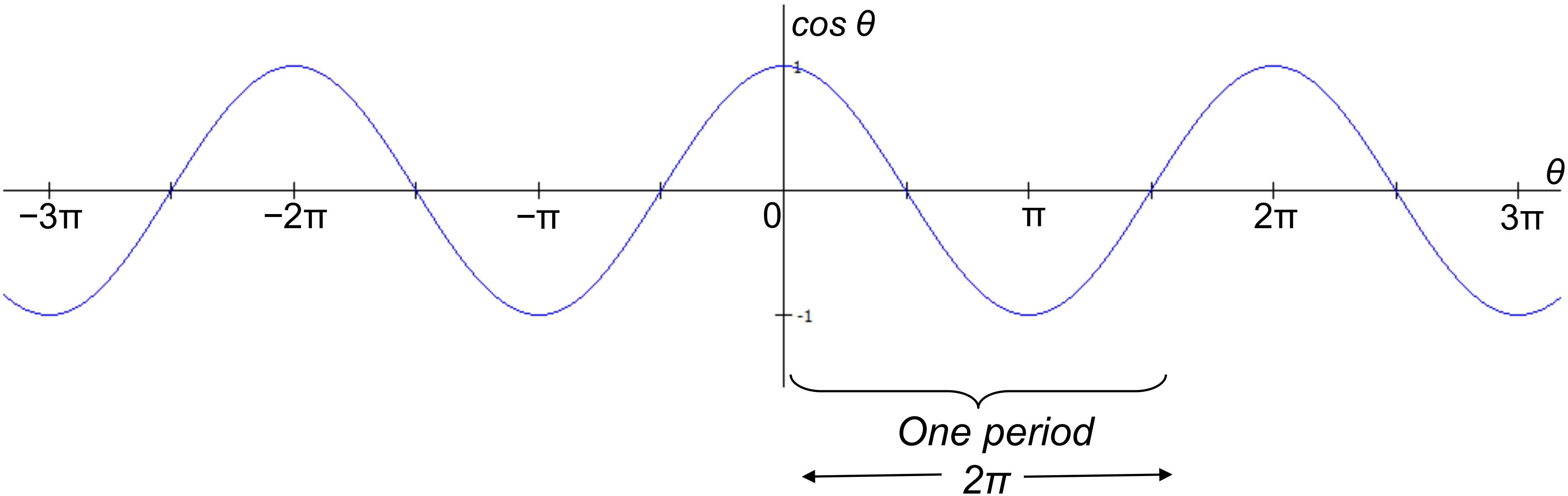
To sketch the graph of $y = \cos x$ first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1

Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.



Cosine is a periodic function: p = 2π



cos θ: Domain (angle measures): all real numbers, (−∞, ∞)
Range (ratio of sides): −1 to 1, inclusive [−1, 1]

cos θ is an **even** function; it is symmetric wrt the y-axis.

$$\forall \theta \in Domain,\ \cos(-\theta) = \cos(\theta)$$

Properties of Sine and Cosine graphs

1. The domain is the set of real numbers
2. The range is set of “y” values such that $-1 \leq y \leq 1$
3. The maximum value is 1 and the minimum value is -1
4. The graph is a smooth curve
5. Each function cycles through all the values of the range over an x interval or 2π
6. The cycle repeats itself identically in both direction of the x-axis

Given : A sin Bx

- Amplitude = IAI
- period = 2π/B

• Sine Graph

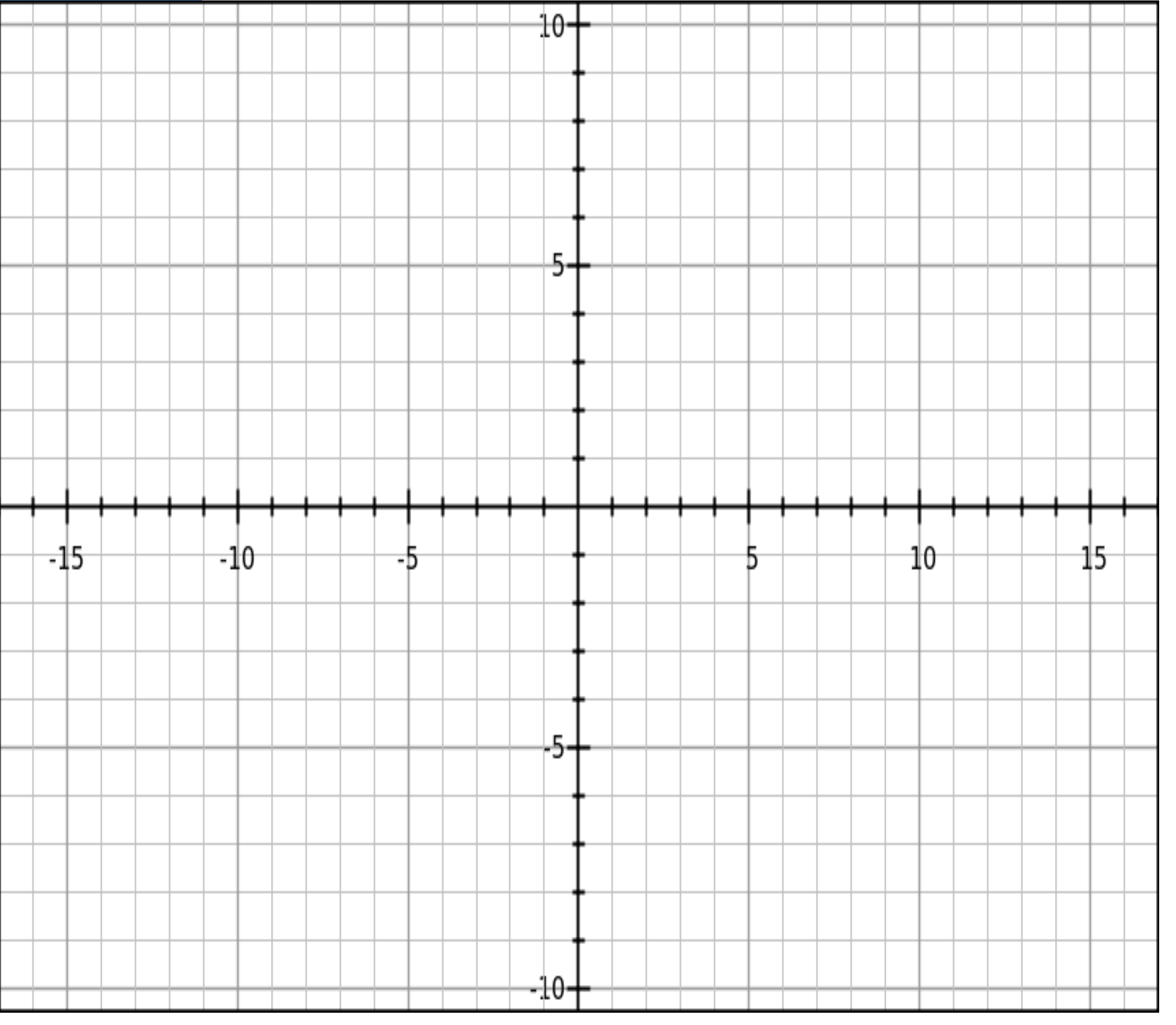
◉ Example:

$y=5\sin 2X$

› Amp=5

› Period=2π/2

= π



•Cosine Graph

Given : A sin Bx

- Amplitude = IAI
- period = 2π/B

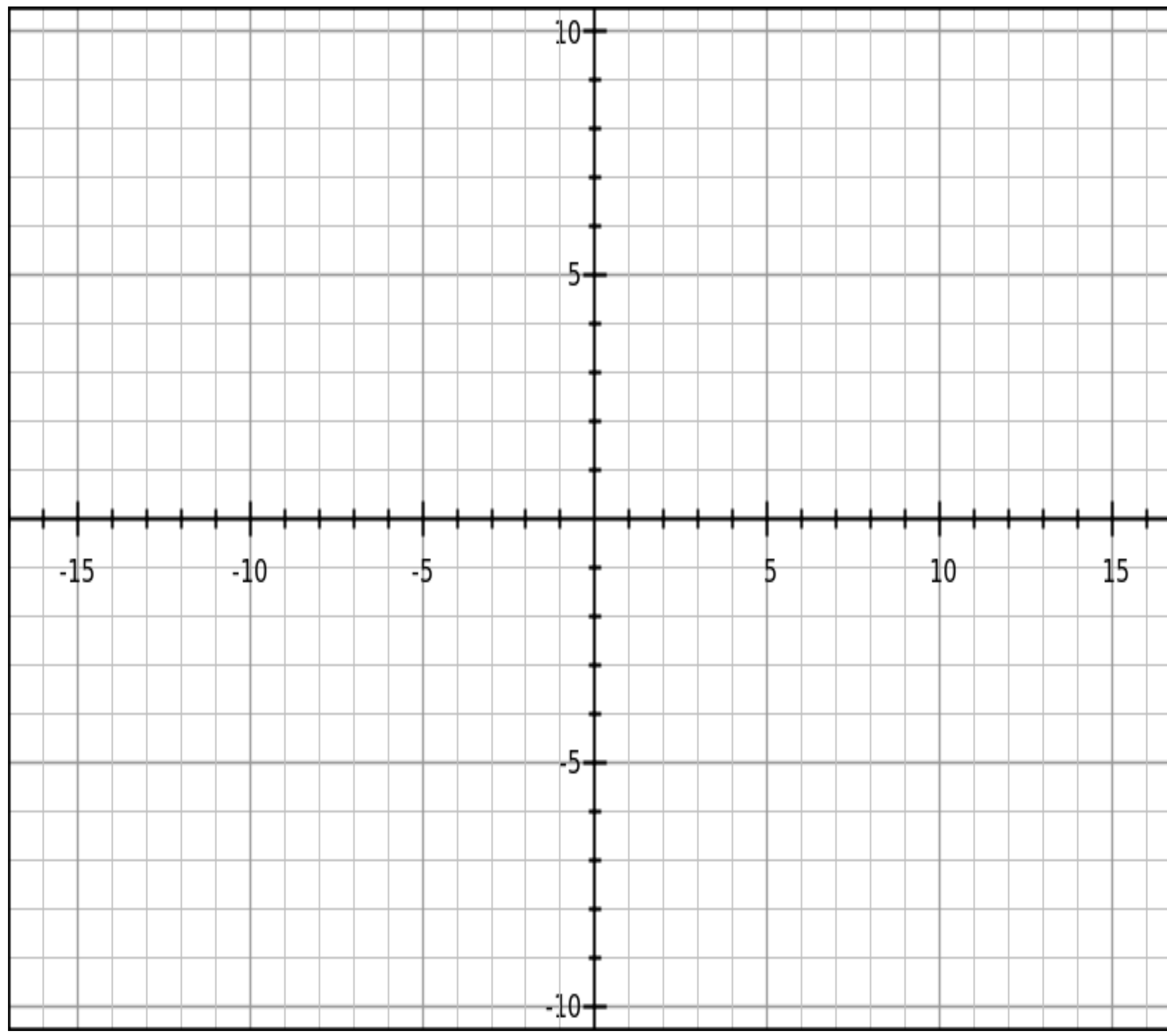
- Example:

  y=2cos 1/2 X

› Amp= 2

› Period= 2π/(1/2)

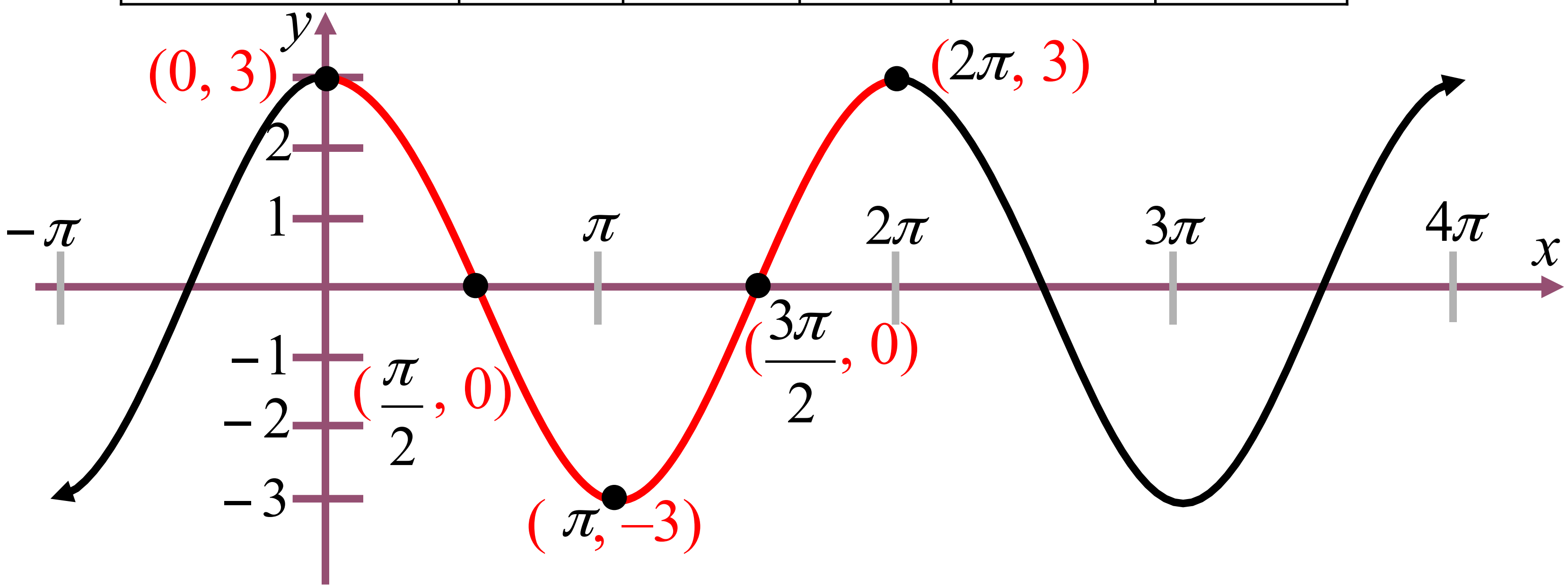
  4 π



Example: Sketch the graph of $y = 3 \cos x$ on the interval $[-\pi, 4\pi]$.

Partition the interval $[0, 2\pi]$ into four equal parts. Find the five key points; graph one cycle; then repeat the cycle over the interval.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = 3 \cos x$	3	0	-3	0	3
	max	x-int	min	x-int	max



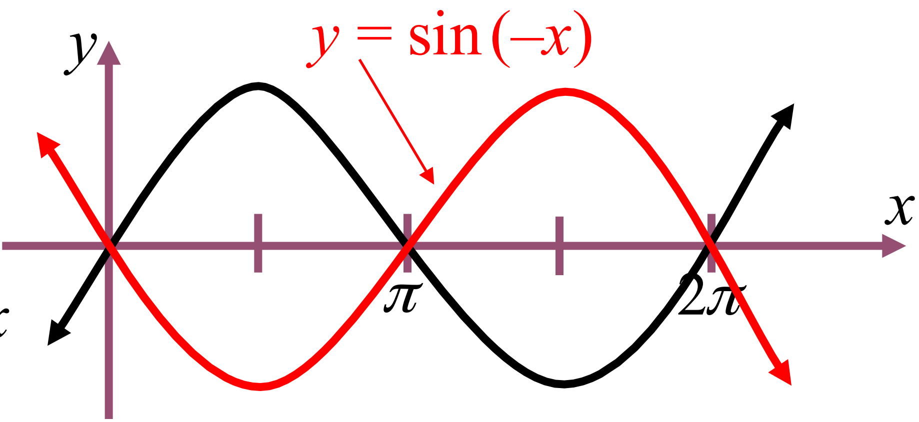
Use basic trigonometric identities to graph $y = f(-x)$

Example : Sketch the graph of $y = \sin(-x)$.

The graph of $y = \sin(-x)$ is the graph of $y = \sin x$ reflected in the x-axis.

Use the identity
$\sin(-x) = -\sin x$

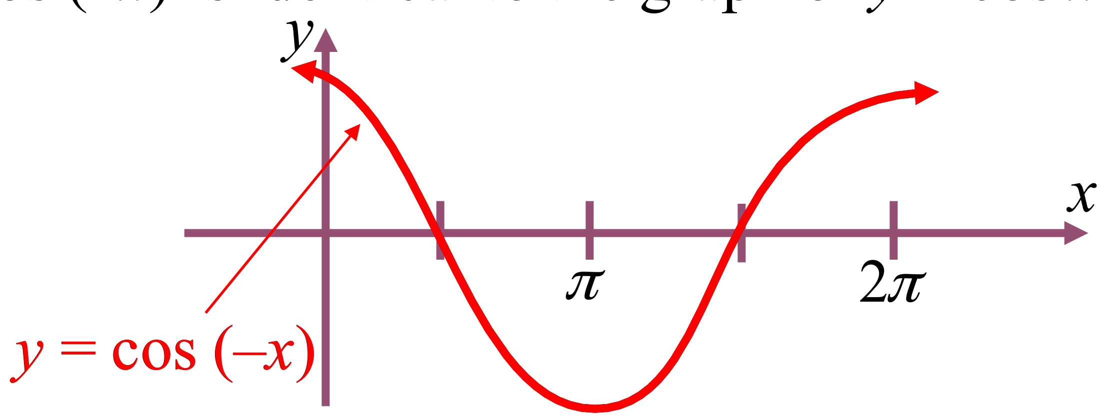
$y = \sin x$



Example : Sketch the graph of $y = \cos(-x)$.

The graph of $y = \cos(-x)$ is identical to the graph of $y = \cos x$.

Use the identity
$\cos(-x) = \cos x$



Example: Sketch the graph of $y = 2\sin(-3x)$.

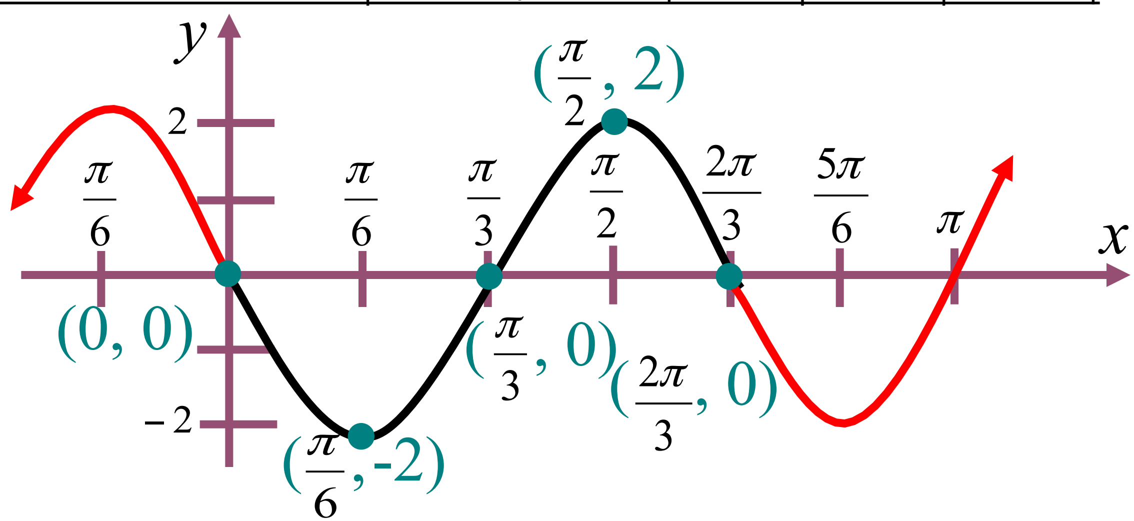
Rewrite the function in the form $y = a \sin bx$ with $b > 0$

Use the identity $\sin(-x) = -\sin x$: $\quad y = 2\sin(-3x) = -2\sin 3x$

amplitude: $|a| = |-2| = 2$ $\qquad$ period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{3}$

Calculate the five key points.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$y = -2\sin 3x$	0	-2	0	2	0



Tangent Function

Recall that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

Since *cos θ* is in the denominator, when *cos θ* = 0, *tan θ* is undefined.
This occurs at π intervals, offset by π/2: { … −π/2, π/2, 3π/2, 5π/2, … }

Let's create an x/y table from *θ* = −π/2 to *θ* = π/2 (one π interval),
with 5 input angle values.

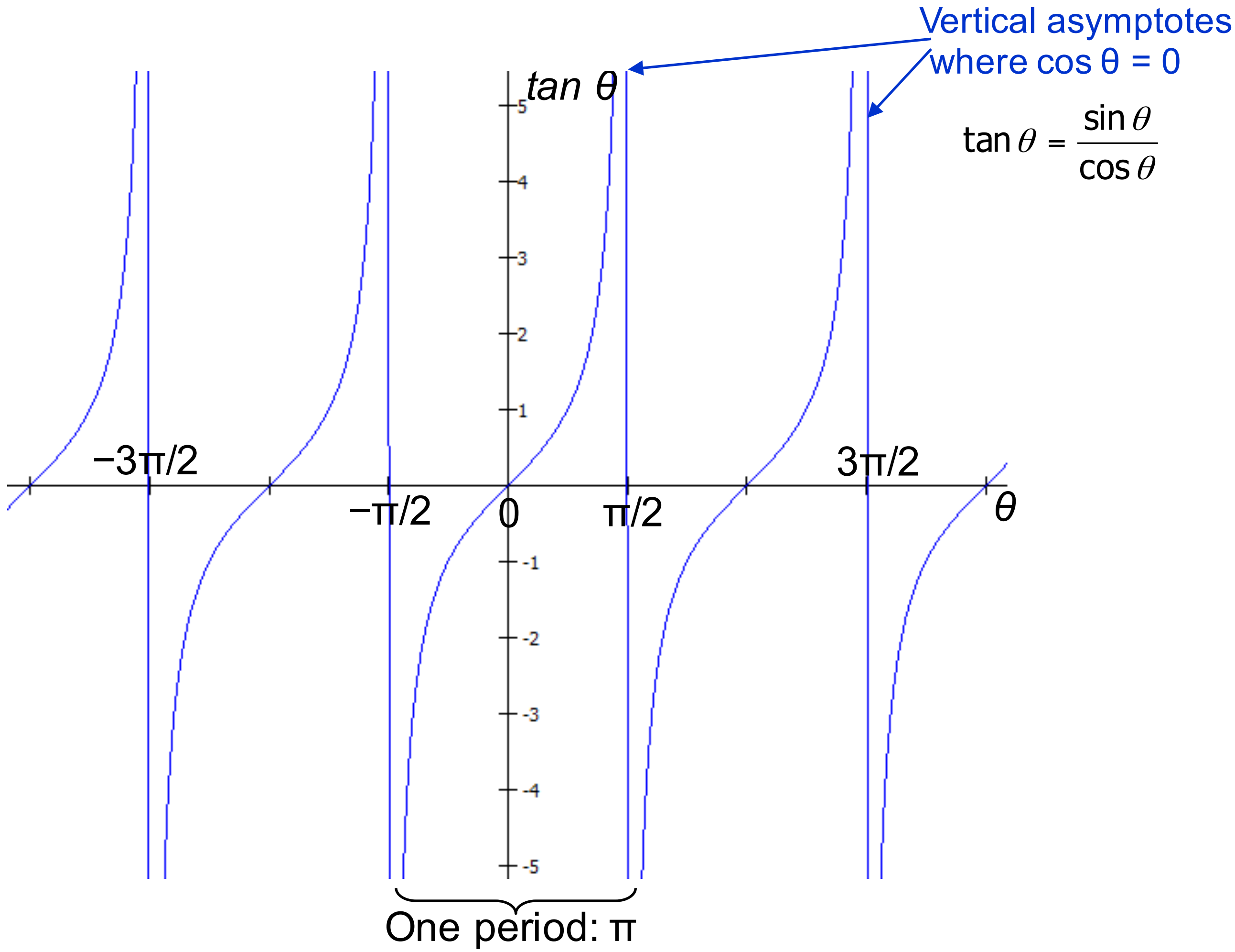
θ	***sin θ***	***cos θ***	***tan θ***
−π/2	−1	0	und
−π/4	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	−1
0	0	1	0
π/4	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
π/2	1	0	und

→

θ	***tan θ***
−π/2	und
−π/4	−1
0	0
π/4	1
π/2	und

Graph of Tangent Function: Periodic

θ	$\tan\theta$
−π/2	Und (-∞)
−π/4	−1
0	0
π/4	1
π/2	Und(∞)



tan θ: Domain (angle measures): θ ≠ π/2 + π*n*
Range (ratio of sides): all real numbers (−∞, ∞)

tan θ is an **odd** function; it is symmetric wrt the origin.

$$\forall \theta \in Domain,\ \tan(-\theta) = -\tan(\theta)$$

Graph of the Tangent Function

To graph $y = \tan x$, use the identity $\tan x = \dfrac{\sin x}{\cos x}$.

At values of x for which $\cos x = 0$, the tangent function is undefined and its graph has vertical asymptotes.

Properties of $y = \tan x$

1. Domain : all real x

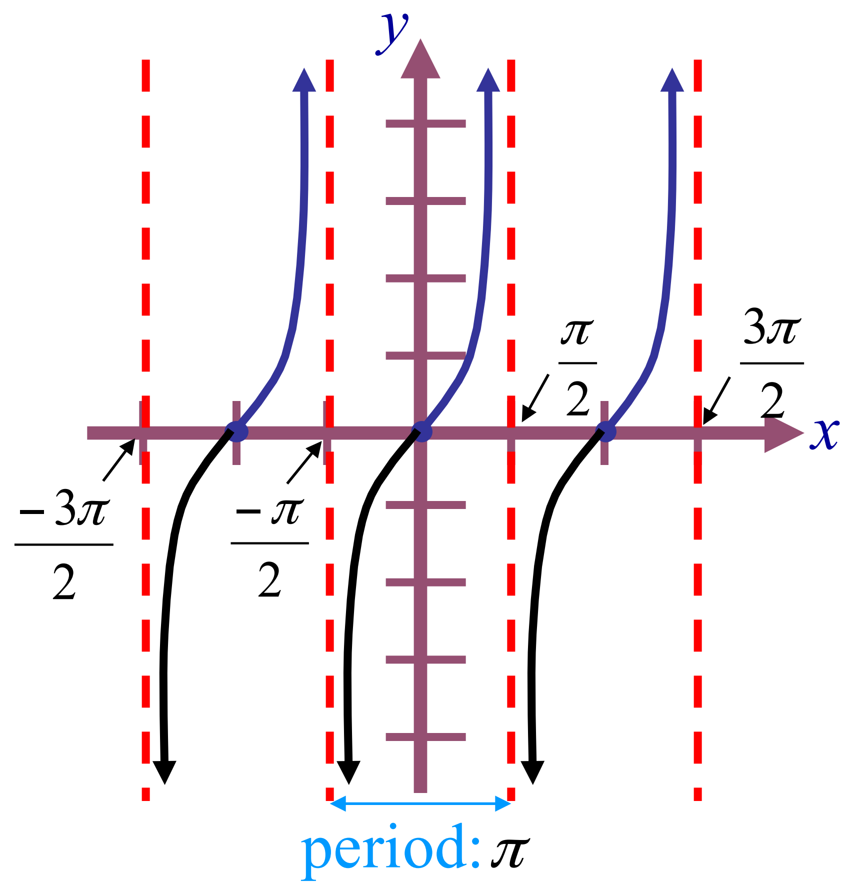
   $x \neq k\pi + \dfrac{\pi}{2} \ (k \in Z)$

2. Range: $(-\infty, +\infty)$

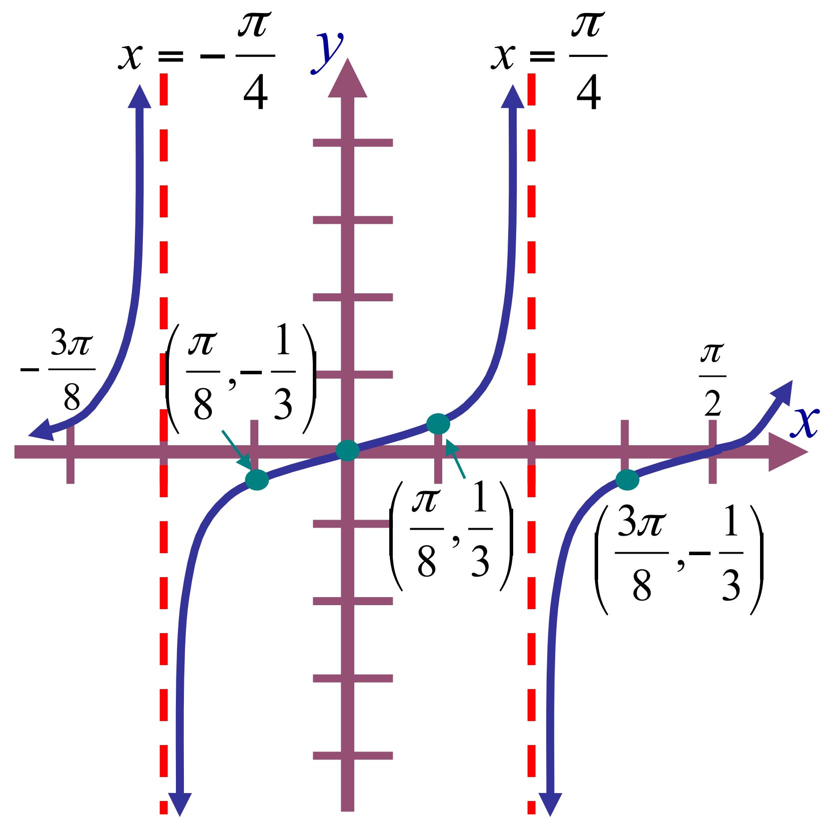
3. Period: π

4. Vertical asymptotes:

   $x = k\pi + \dfrac{\pi}{2} \ (k \in Z)$



Example: Find the period and asymptotes and sketch the graph of $y = \frac{1}{3}\tan 2x$



1. Period of $y = \tan x$ is π .
   $\rightarrow$ Period of $y = \tan 2x$ is $\frac{\pi}{2}$.

2. Find consecutive vertical asymptotes by solving for x:

$$2x = -\frac{\pi}{2}, 2x = \frac{\pi}{2}$$

Vertical asymptotes: $x = -\frac{\pi}{4}, x = \frac{\pi}{4}$

3. Plot several points in $(0, \frac{\pi}{2})$

4. Sketch one branch and repeat.

x	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{3\pi}{8}$
$y = \frac{1}{3}\tan 2x$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$

Cotangent Function

Recall that $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

Since *sin θ* is in the denominator, when *sin θ* = 0, *cot θ* is undefined.

This occurs @ π intervals, starting at 0: { … −π, 0, π, 2π, … }

Let's create an x/y table from θ = 0 to θ = π (one π interval),
with 5 input angle values.

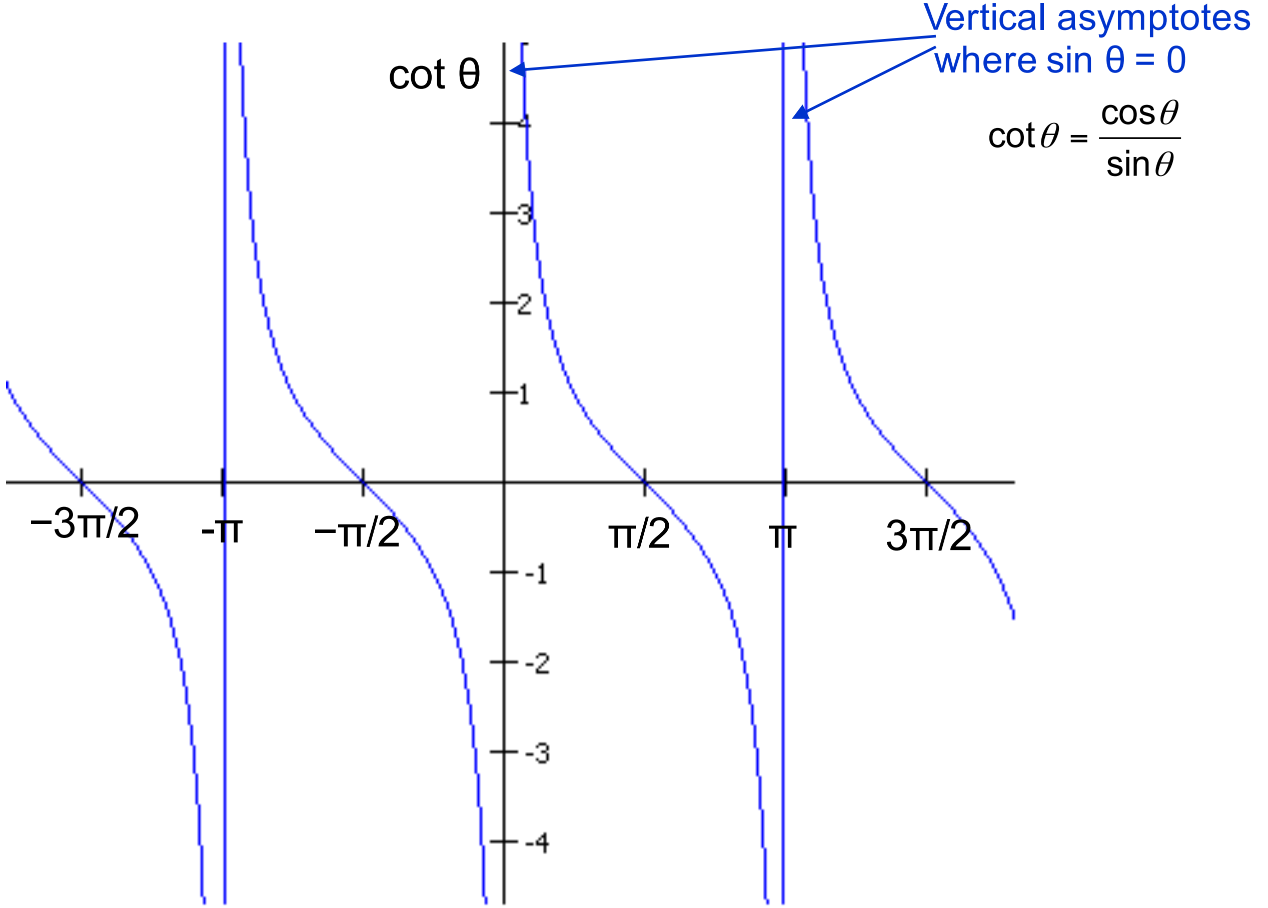
θ	***sin θ***	***cos θ***	***cot θ***
0	0	1	Und ∞
π/4	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
π/2	1	0	0
3π/4	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	−1
π	0	−1	Und−∞

→

θ	***cot θ***
0	Und ∞
π/4	1
π/2	0
3π/4	−1
π	Und−∞

Graph of Cotangent Function: Periodic

θ	$\cot\theta$
0	∞
π/4	1
π/2	0
3π/4	−1
π	−∞



cot θ: Domain (angle measures): θ ≠ π*n*

Range (ratio of sides): all real numbers (−∞, ∞)

cot θ is an **odd** function; it is symmetric wrt the origin.

$\forall\theta\in Domain,\ \ tan(-\theta) = -tan(\theta)$

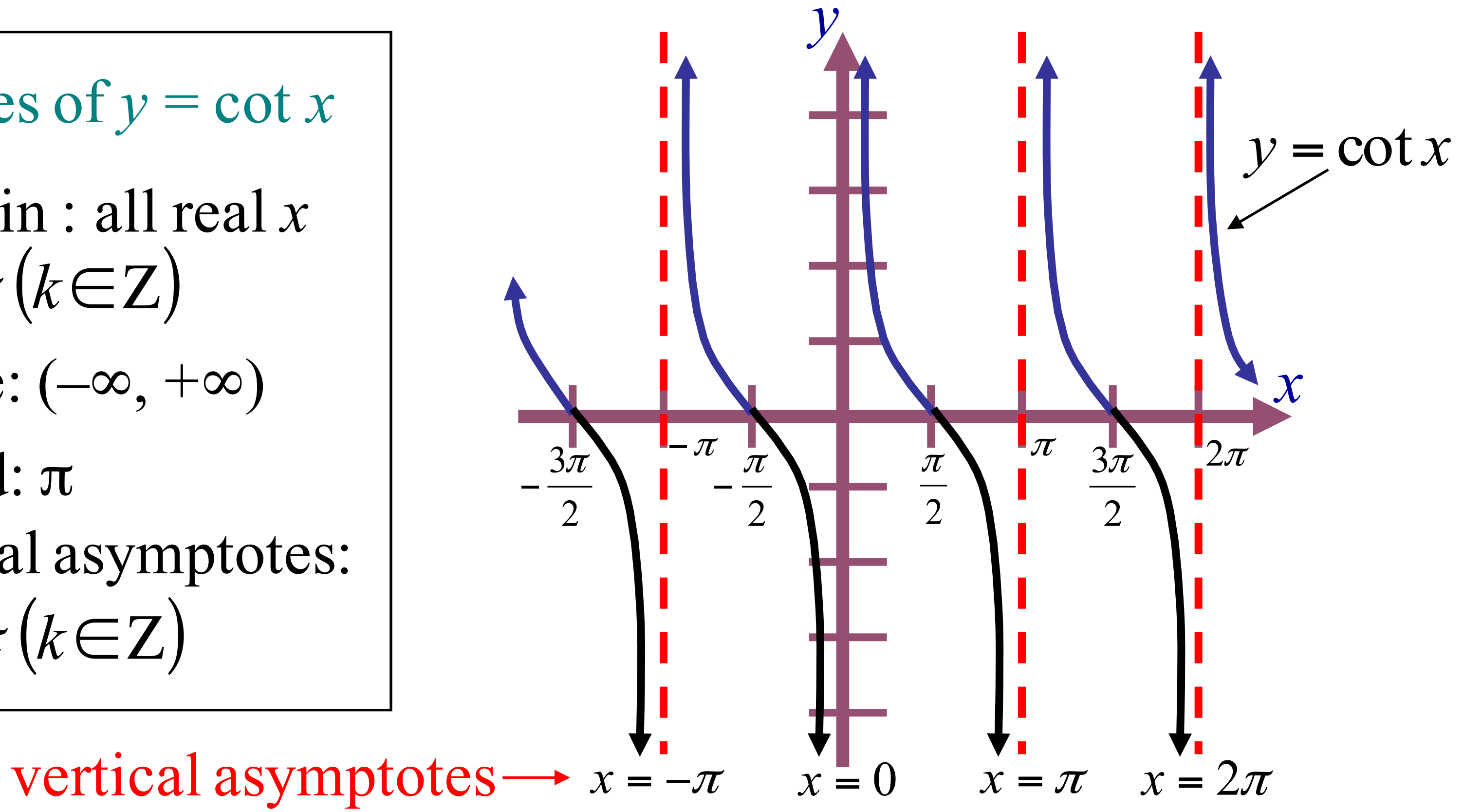
Graph of the Cotangent Function

To graph $y = \cot x$, use the identity $\cot x = \dfrac{\cos x}{\sin x}$.

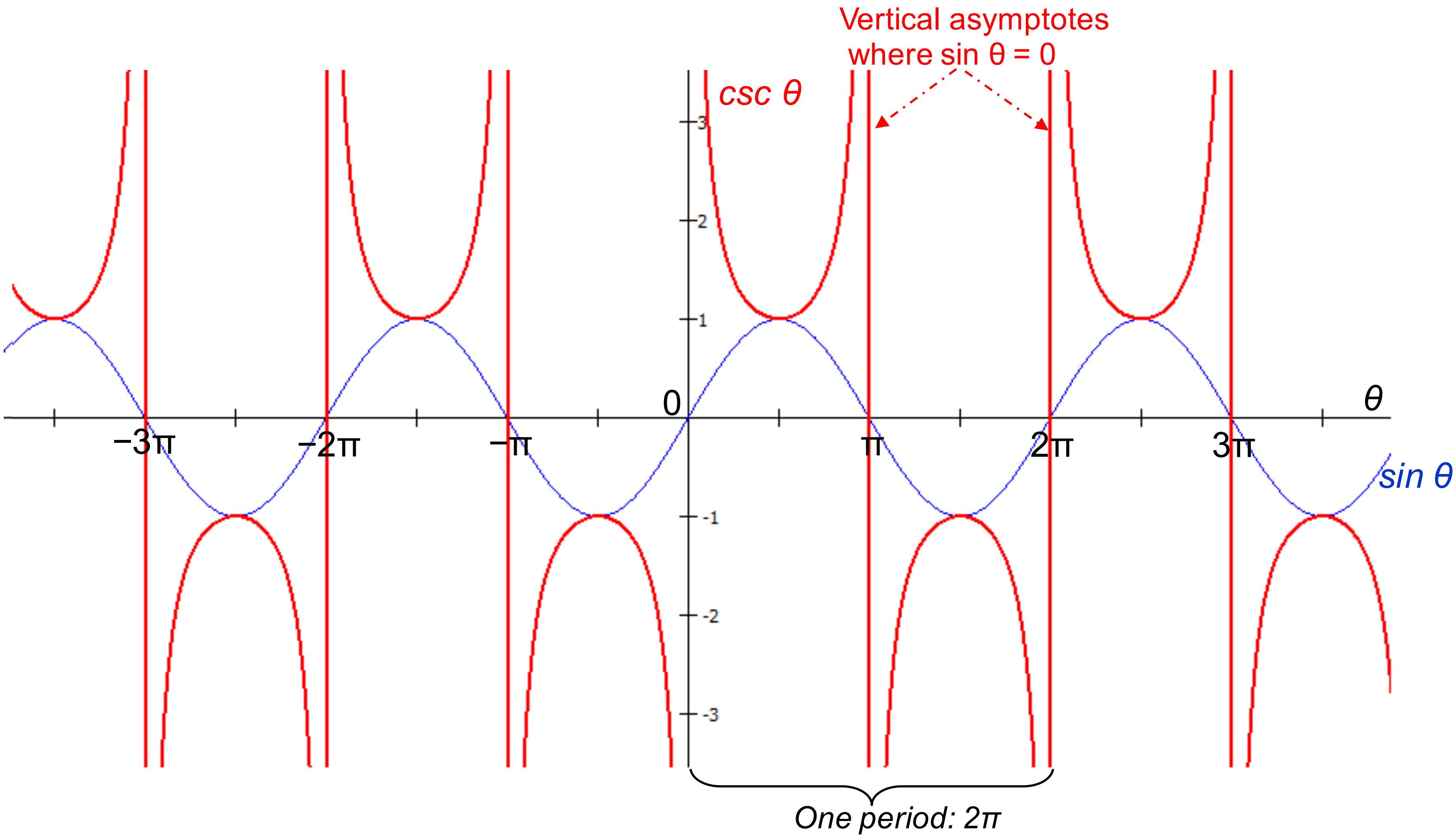
At values of x for which $\sin x = 0$, the cotangent function is undefined and its graph has vertical asymptotes.

Properties of $y = \cot x$

1. Domain : all real x
   $x \neq k\pi \ (k \in Z)$
2. Range: $(-\infty, +\infty)$
3. Period: π
4. Vertical asymptotes:
   $x = k\pi \ (k \in Z)$



Cosecant is the reciprocal of sine



sin θ: Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

csc θ: Domain: $\theta \neq \pi n$
(where $\sin \theta = 0$)
Range: $|\csc \theta| \geq 1$
or $(-\infty, -1] \cup [1, \infty)$

sin θ and *csc θ*
are **odd**
(symm wrt origin)

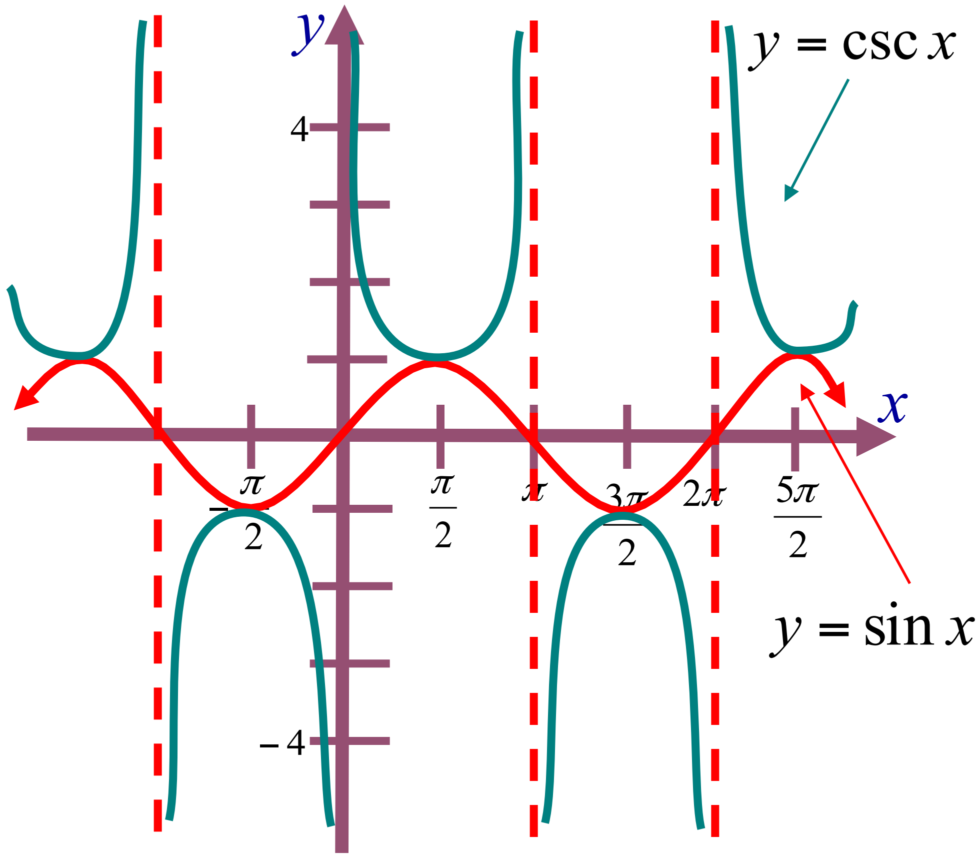
Graph of the Cosecant Function

To graph $y = \csc x$, use the identity $\csc x = \dfrac{1}{\sin x}$.

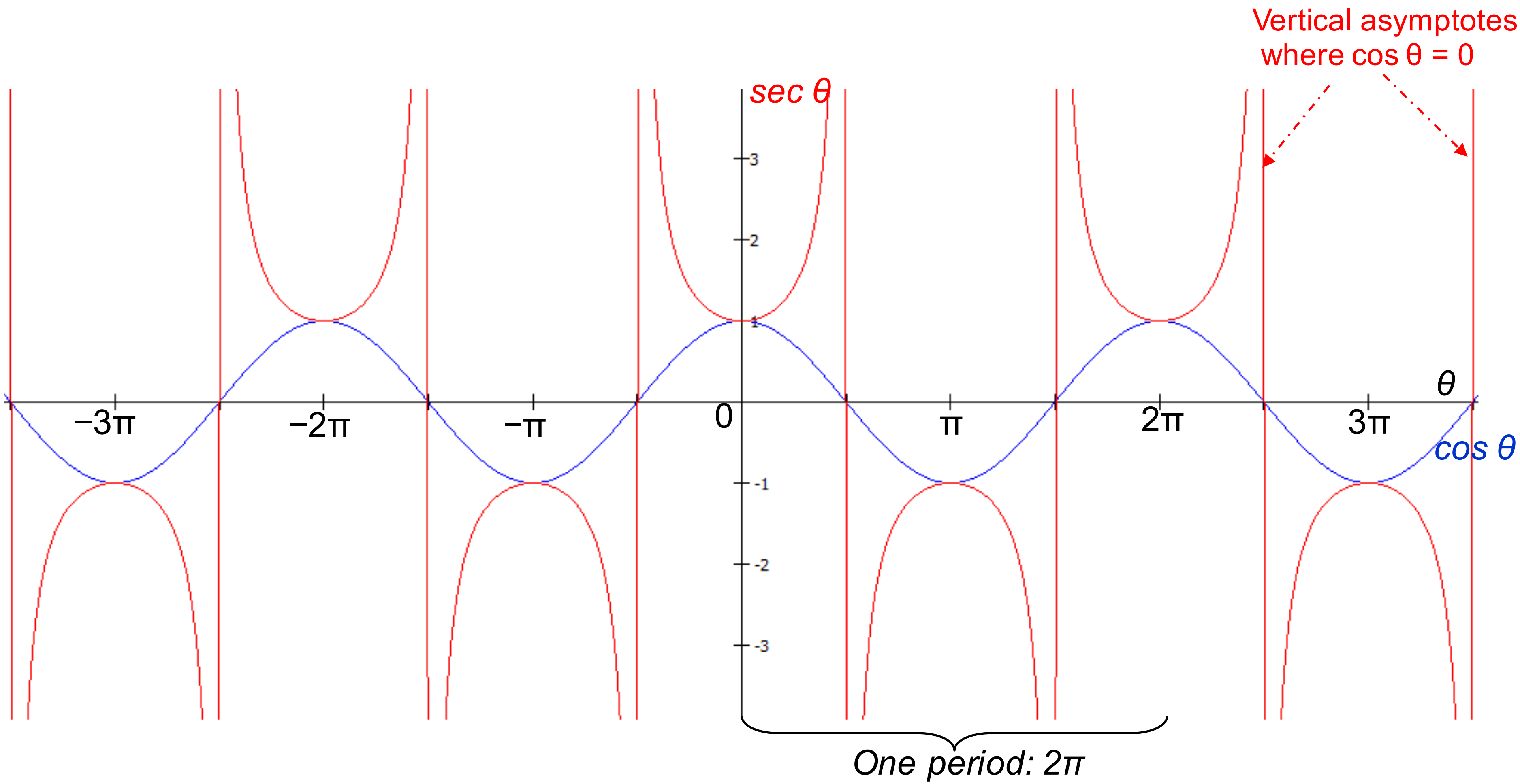
At values of x for which $\sin x = 0$, the cosecant function is undefined and its graph has vertical asymptotes.

Properties of $y = \csc x$

1. domain : all real x
   $x \neq k\pi \left(k \in Z\right)$
2. range: $(-\infty, -1] \cup [1, +\infty)$
3. period: π
4. vertical asymptotes:
   $x = k\pi \left(k \in Z\right)$
   where sine is zero.



Secant is the reciprocal of cosine



cos θ: Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

sec θ: Domain: $\theta \neq \pi/2 + \pi n$
(where $\cos\theta = 0$)
Range: $|\sec\theta| \geq 1$
or $(-\infty, -1] \cup [1, \infty)$

cos *θ* and sec *θ* are **even** (symm wrt y-axis)

Graph of the Secant Function

The graph $y = \sec x$, use the identity $\sec x = \dfrac{1}{\cos x}$.

At values of x for which $\cos x = 0$, the secant function is undefined and its graph has vertical asymptotes.

Properties of $y = \sec x$

1. domain : all real x

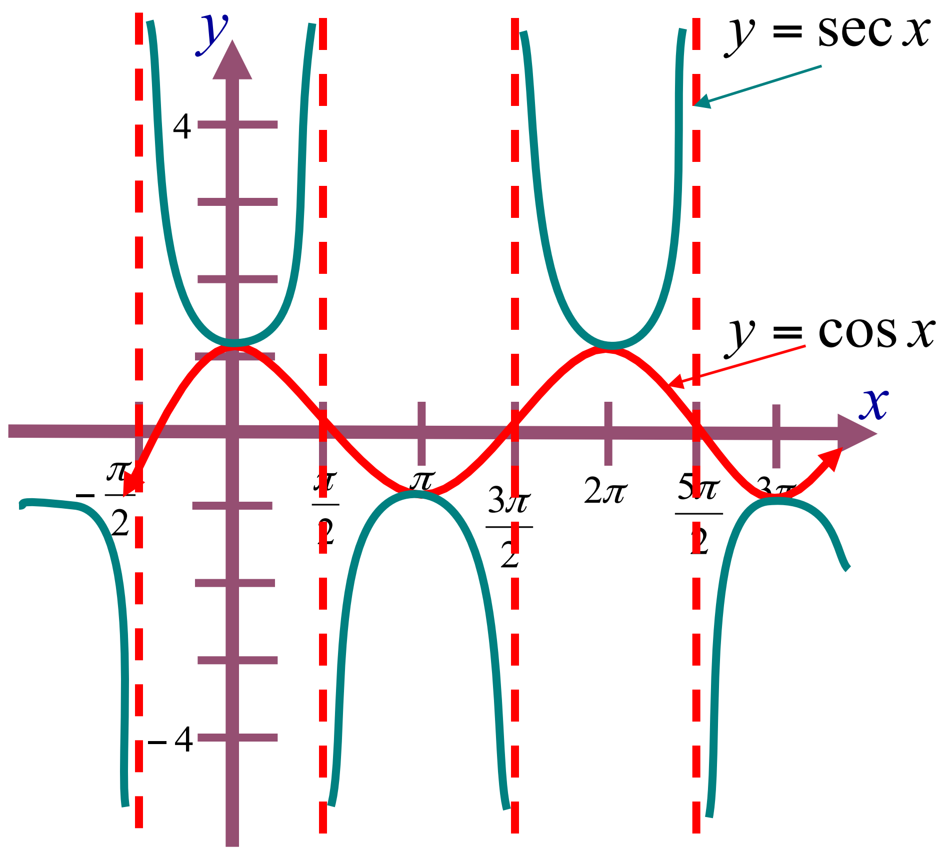
   $x \neq k\pi + \dfrac{\pi}{2} \ (k \in Z)$

2. range: $(-\infty, -1] \cup [1, +\infty)$

3. period: π

4. vertical asymptotes:

   $x = k\pi + \dfrac{\pi}{2} \ (k \in Z)$



Summary of Graph Characteristics

	Def'n		Period	Domain	Range	Even/Odd
	Δ	O				
sin θ						
csc θ						
cos θ						
sec θ						
tan θ						
cot θ						

Summary of Graph Characteristics

	Def'n		Period	Domain	Range	Even/Odd
	Δ	O				
sin θ	$\frac{opp}{hyp}$	$\frac{y}{r}$	2π	(−∞, ∞)	−1 ≤ x ≤ 1 or [−1, 1]	odd
csc θ	$\frac{1}{\sin\theta}$	$\frac{r}{y}$	2π	$\theta \neq \pi n$	$\lvert \csc\theta \rvert \geq 1$ or (−∞, −1] ∪ [1, ∞)	odd
cos θ	$\frac{adj}{hyp}$	$\frac{x}{r}$	2π	(−∞, ∞)	All Reals or (−∞, ∞)	even
sec θ	$\frac{1}{\sin\theta}$	$\frac{r}{y}$	2π	$\theta \neq \pi 2 + \pi n$	$\lvert \sec\theta \rvert \geq 1$ or (−∞, −1] ∪ [1, ∞)	even
tan θ	$\frac{\sin\theta}{\cos\theta}$	$\frac{y}{x}$	π	$\theta \neq \pi 2 + \pi n$	All Reals or (−∞, ∞)	odd
cot θ	$\frac{\cos\theta}{\sin\theta}$	$\frac{x}{y}$	π	$\theta \neq \pi n$	All Reals or (−∞, ∞)	odd

14. 2: Translations of Trigonometric Graphs

• Without looking at your notes, try to sketch the basic shape of each trig function:

1) Sine:

2) Cosine:

3) Tangent:

More Transformations

- We have seen two types of transformations on trig graphs: vertical stretches and horizontal stretches.

- There are three more: vertical translations (slides), horizontal translations, and reflections (flips).

More Transformations

- Here is the full general form for the sine function:

$$y = k + a\sin b(x - h)$$

- Just as with parabolas and other functions, h and k are translations:
- h slides the graph horizontally (opposite of sign)
- k slides the graph vertically
- Also, if a is negative, the graph is flipped vertically.

More Transformations

- ***To graph a sine or cosine graph:***

1. Graph the original graph with the correct amplitude and period (like section 14.1).

2. Translate h units horizontally and k units vertically.

3. Reflect vertically at its new position if a is negative (or reflect first, then translate).

Examples

➢ Describe how each graph would be transformed:

1. $y = 2 + \sin x$

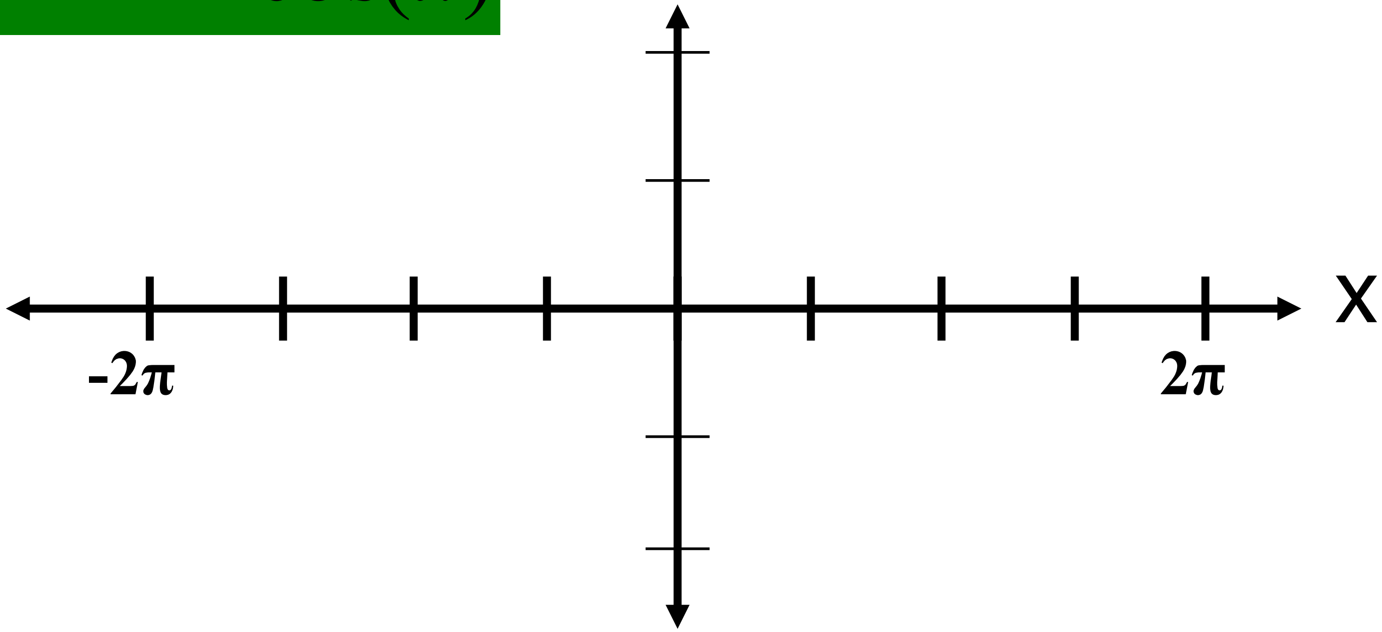
2. $y = \cos\left(x + \frac{\pi}{2}\right)$

3. $y = -2 - \sin(x - \pi)$

Examples

- State the amplitude and period, then graph:

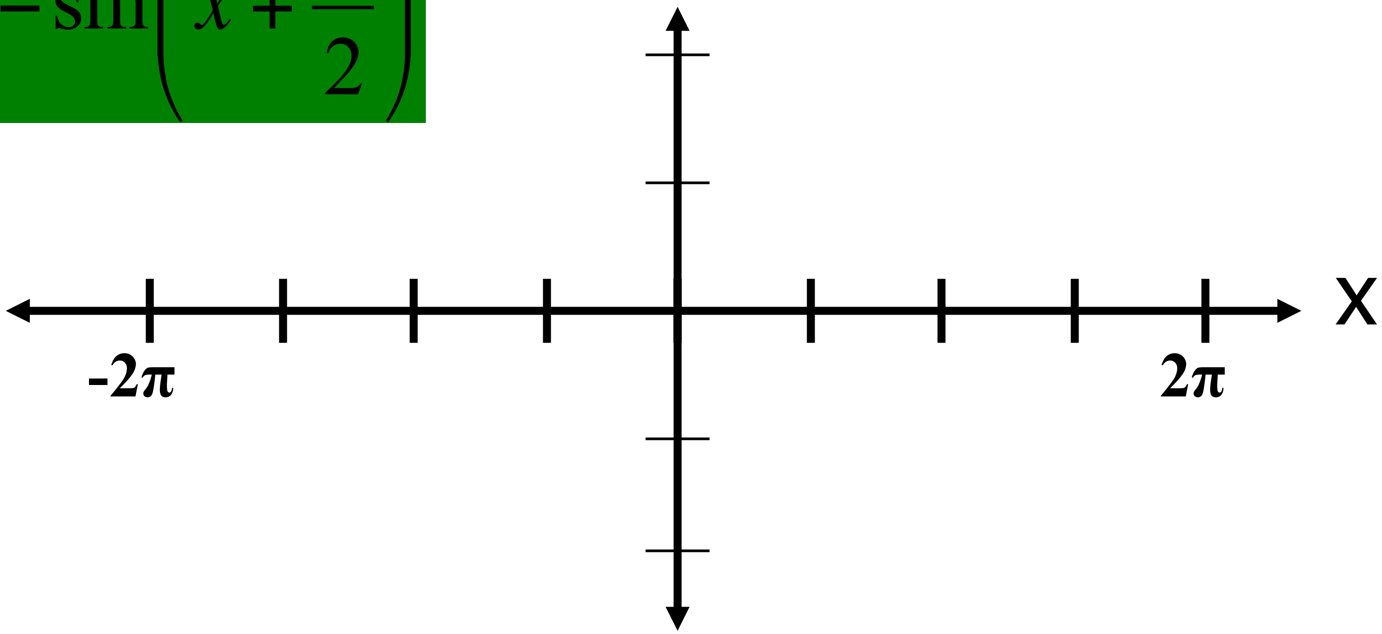
$$y = -2 + \cos(x)$$



Examples

- State the amplitude and period, then graph:

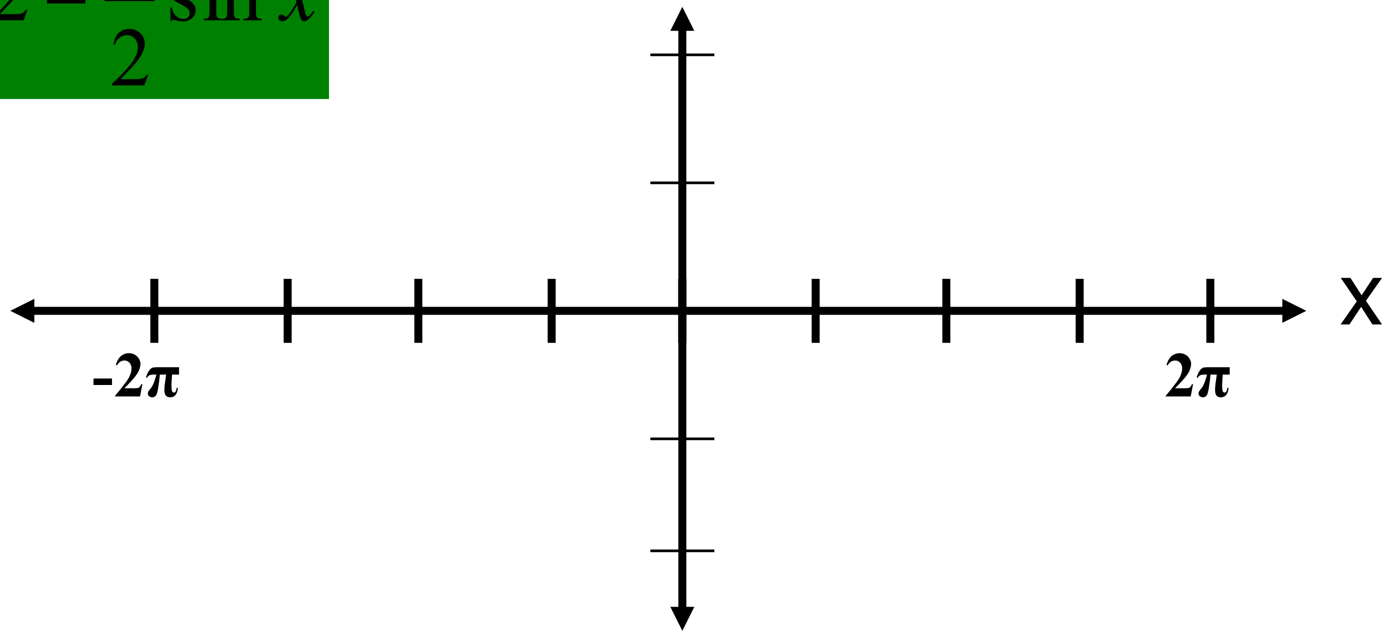
$$y = -\sin\left(x + \frac{\pi}{2}\right)$$



Examples

- State the amplitude and period, then graph:

$$y = 2 - \frac{1}{2}\sin x$$



Examples

- Write an equation of the graph described:
- The graph of y = cos x translated up 3 units, right π units, and reflected vertically.

14.3: trigonometric Identities

- Reciprocal Identities
- Quotient Identities
- Pythagorean Identities
- Opposite Angles Identity

Key Vocabulary

1. **Identity:** a statement of equality between two expressions that is true for *all* values of the variable(s)
2. **Trigonometric Identity:** an identity involving trigonometric expressions
3. **Counterexample:** an example that shows an equation is false.

Prove that sin(x)tan(x) = cos(x) is not a trig identity by producing a counterexample.

- You can do this by picking almost any angle measure.
- Use ones that you know exact values for:

  → 0, π/6, π/4, π/3, π/2, and π

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

(or $\sin\theta\csc\theta = 1$ $\qquad$ $\cos\theta\sec\theta = 1$ $\qquad$ $\tan\theta\cot\theta = 1$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Why?

Do you remember the Unit Circle?

- What is the equation for the unit circle?

$$x^2 + y^2 = 1$$

- **What does x = ? What does y = ?**

  (in terms of trig functions)

$$\sin^2\theta + \cos^2\theta = 1$$

Pythagorean Identity!

Take the Pythagorean Identity and discover a new one!

Hint: Try dividing everything by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}.$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Quotient Identity

another Pythagorean Identity

Reciprocal Identity

Take the Pythagorean Identity and discover a new one!

Hint: Try dividing everything by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}.$$

$$1 + \cot^2\theta = \csc^2\theta$$

Quotient Identity

Reciprocal Identity

a third Pythagorean Identity

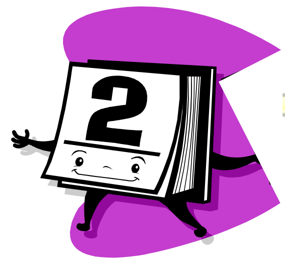
Opposite Angle Identities

sometimes these are called even/odd identities

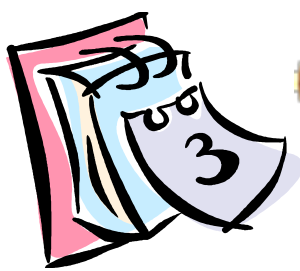
$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

Simplify each expression.

$$\frac{\csc\theta}{\cot\theta}$$



$$\cos x \csc x \tan x$$



$$\cos x \cot x + \sin x$$

Using the identities you now know, find the trig value.

1 If cosθ = 3/4,
find secθ.

$$0° < \theta < 90°$$

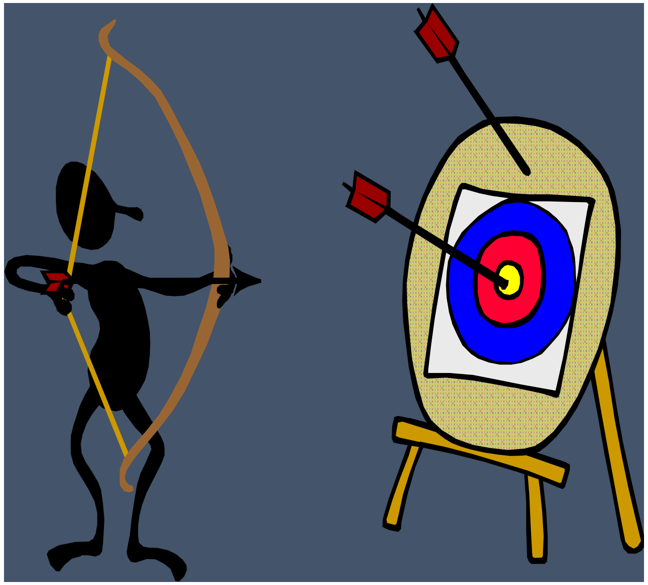
2 θ = 3/5,
find cscθ.

$$\frac{3\pi}{2} < \theta < 2\pi$$

3 $\sin\theta = -1/3$, $180^o < \theta < 270^o$; find $\tan\theta$

4 $\sec\theta = -7/5$, $\pi < \theta < 3\pi/2$; find $\sin\theta$

– Similarities and Differences



a) How do you find the amplitude and period for sine and cosine functions?

b) How do you find the amplitude, period and asymptotes for tangent?

c) What process do you follow to graph any of the trigonometric functions?