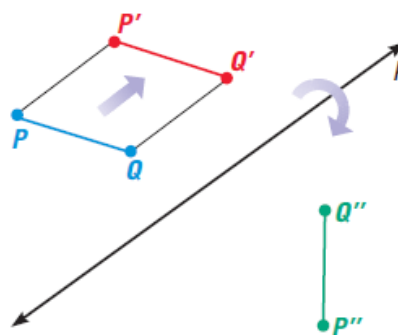


Notes 10.5 Glide Reflections and Compositions

GOAL 1 USING GLIDE REFLECTIONS

A translation, or glide, and a reflection can be performed one after the other to produce a transformation known as a *glide reflection*. A **glide reflection** is a transformation in which every point P is mapped onto a point P'' by the following steps:

1. A translation maps P onto P' .
2. A reflection in a line k parallel to the direction of the translation maps P' onto P'' .



As long as the line of reflection is parallel to the direction of the translation, it does not matter whether you glide first and then reflect, or reflect first and then glide.

EXAMPLE 1 Finding the Image of a Glide Reflection

Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

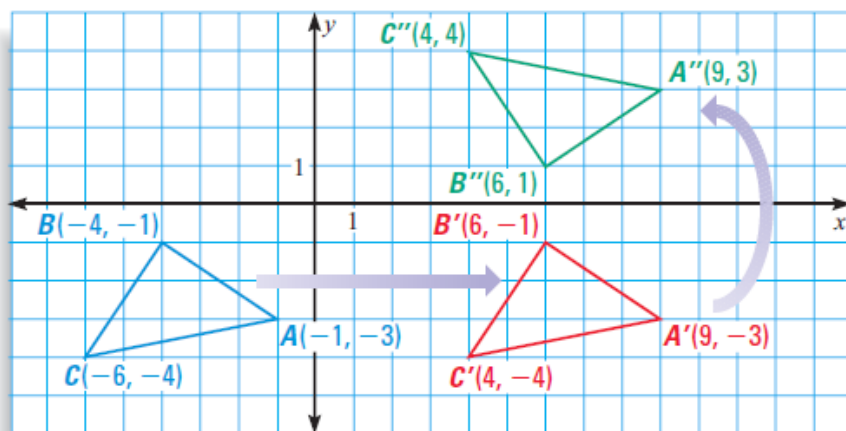
$$A(-1, -3), B(-4, -1), C(-6, -4)$$

$$\text{Translation: } (x, y) \rightarrow (x + 10, y)$$

Reflection: in the x -axis

SOLUTION

Begin by graphing $\triangle ABC$. Then, shift the triangle 10 units to the right to produce $\triangle A'B'C'$. Finally, reflect the triangle in the x -axis to produce $\triangle A''B''C''$.



In Example 1, try reversing the order of the transformations. Notice that the resulting image will have the same coordinates as $\triangle A''B''C''$ above. This is true because the line of reflection is parallel to the direction of the translation.

Notes 10.5 Glide Reflections and Compositions Continued

GOAL 2 USING COMPOSITIONS

When two or more transformations are combined to produce a single transformation, the result is called a **composition** of the transformations.

EXAMPLE 2 Finding the Image of a Composition

Sketch the image of \overline{PQ} after a composition of the given rotation and reflection.

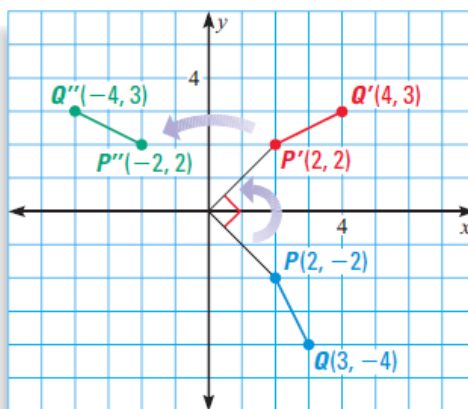
$$P(2, -2), Q(3, -4)$$

Rotation: 90° counterclockwise about the origin

Reflection: in the y -axis

SOLUTION

Begin by graphing \overline{PQ} . Then rotate the segment 90° counterclockwise about the origin to produce $\overline{P'Q'}$. Finally, reflect the segment in the y -axis to produce $\overline{P''Q''}$.



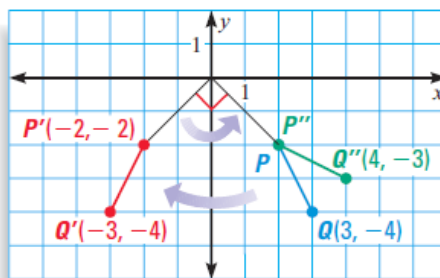
EXAMPLE 3 Comparing Orders of Compositions

Repeat Example 2, but switch the order of the composition by performing the reflection first and the rotation second. What do you notice?

SOLUTION

Graph \overline{PQ} . Then reflect the segment in the y -axis to obtain $\overline{P'Q'}$. Rotate $\overline{P'Q'}$ 90° counterclockwise about the origin to obtain $\overline{P''Q''}$. Instead of being in Quadrant II, as in Example 2, the image is in Quadrant IV.

- ▶ The order which the transformations are performed affects the final image.



EXAMPLE 4 Describing a Composition

Describe the composition of transformations in the diagram.

SOLUTION

Two transformations are shown. First, figure $ABCD$ is reflected in the line $x = 2$ to produce figure $A'B'C'D'$. Then, figure $A'B'C'D'$ is rotated 90° clockwise about the point $(2, 0)$ to produce figure $A''B''C''D''$.

