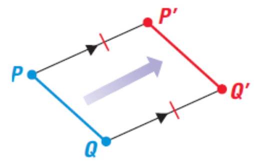
Notes 10.1 Translations continued

A **translation** is a transformation that maps every two points P and Q in the plane to points P' and Q', so that the following properties are true:



1. $PP' = QQ'$
2. $\overline{PP'} \parallel \overline{QQ'}$, or $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

THEOREM

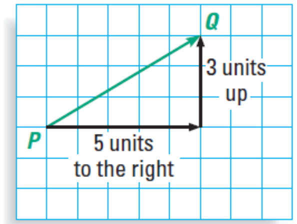
THEOREM 7.4 ***Translation Theorem***

A translation is an isometry.

GOAL 2 TRANSLATIONS USING VECTORS

Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude*, or size, and is represented by an arrow drawn between two points.

The diagram shows a vector. The **initial point**, or starting point, of the vector is P and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The *horizontal component* of $\overrightarrow{PQ}$ is 5 and the *vertical component* is 3.

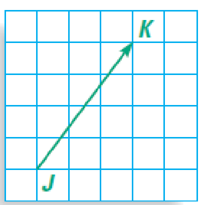


The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3\rangle$.

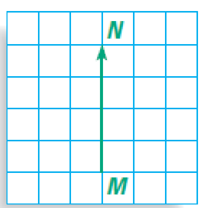
EXAMPLE 3 *Identifying Vector Components*

In the diagram, name each vector and write its component form.

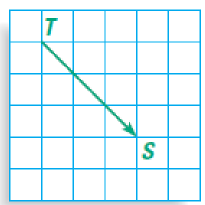
a.



b.



c.



SOLUTION

a. The vector is $\overrightarrow{JK}$. To move from the initial point J to the terminal point K, you move 3 units to the right and 4 units up. So, the component form is $\langle 3, 4\rangle$.

b. The vector is $\overrightarrow{MN} = \langle 0, 4\rangle$.

c. The vector is $\overrightarrow{TS} = \langle 3, -3\rangle$.

EXAMPLE 4 *Translation Using Vectors*

The component form of $\overrightarrow{GH}$ is $\langle 4, 2\rangle$. Use $\overrightarrow{GH}$ to translate the triangle whose vertices are $A(3, -1)$, $B(1, 1)$, and $C(3, 5)$.

SOLUTION

First graph $\triangle ABC$. The component form of $\overrightarrow{GH}$ is $\langle 4, 2\rangle$, so the image vertices should all be 4 units to the right and 2 units up from the preimage vertices. Label the image vertices as $A'(7, 1)$, $B'(5, 3)$, and $C'(7, 7)$. Then, using a straightedge, draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage to image vertices are parallel.